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Fermionic isocurvature perturbations

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Isocurvature perturbations in the inflationary literature typically involve quantum fluctuations of bosonic field degrees of freedom. In this work, we consider isocurvature perturbations from fermionic quantum fluctuations during inflation. When a stable massive fermion is coupled to a nonconformal sector different from the scalar metric perturbations, observably large amplitude scale invariant isocurvature perturbations can be generated. In addition to the computation of the isocurvature two-point function, an estimate of the local non-Gaussianities is also given and found to be promising for observations in a corner of the parameter space. The results provide a new class of cosmological probes for theories with stable massive fermions. On the technical side, we explicitly renormalize the composite operator in curved spacetime and show that gravitational Ward identities play an important role in suppressing certain contributions to the fermionic isocurvature perturbations.

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I. INTRODUCTION

The cosmic microwave background (CMB) measurements [1–4] and the large scale structure observations [5,6] are consistent with single field inflationary models which can seed approximately adiabatic, scale-invariant, and Gaussian primordial density perturbations [7–15]. However, from the multifield nature of the Standard Model (SM) of particle physics, one may naturally guess that there would be more than one light degrees of freedom during inflation which may be responsible for generating isocurvature primordial perturbation initial conditions. Indeed, in any slow-roll inflationary scenario, noninflaton degrees of freedom must eventually turn on in order to reheat successfully.[1] Hence, isocurvature scenarios are theoretically well motivated.

Isocurvature perturbations have been studied in various scenarios, such as double inflation [16–18], curvaton [19–21], axions [22–27] and gravitationally produced superheavy dark matter [28–31]. Isocurvature perturbations also can generate rich density perturbation phenomenology. For example, unlike standard single field inflationary scenarios, isocurvature perturbations are able to generate large primordial local non-Gaussianities [28,31–45]. However, most previous studies of isocurvature perturbations focused on bosonic degrees of freedom such as axions and curvatons. Fermionic isocurvature degrees of freedom such as gravitinos were only discussed in the literature associated with the decay products of the inflaton or other scalars [46–49]. Furthermore, these fermions discussed in the literature were characterized only by their dependence on the entropy temperature fluctuation δT which was assumed to be directly linked to the curvature perturbation ζ, in a manner consistent with the "separate universe" picture of δN formalism [50]. Such previously discussed fermionic isocurvature scenarios lead to strong correlation or anticorrelation with the curvature perturbation ζ. One can intuitively characterize these previous fermionic isocurvature works as having no fermionic quantum fluctuation information from the inflationary era.

In contrast, we examine in this paper a fermionic isocurvature scenario that is not (significantly) correlated with ζ and has fermionic quantum fluctuation information during inflation encoded in the isocurvature correlator. In our scenario, the horizon length scale interaction dynamics of the fermion particles is important, in sharp contrast with the separate universe picture of δN formalism. As we will show, although classical gravitational field interactions alone are sufficient to generate enough fermions during the exit process of inflation to saturate the phenomenologically required cold dark matter abundance [51,52], fermion propagators in the classical Friedmann-Robertson-Walker (FRW) background is insufficient to produce any observable isocurvature perturbations because of the fact that massless fermions enjoy a classical conformal symmetry.[2] Hence, any large fermion isocurvature correlator must involve couplings to a conformal symmetry breaking sector.

For illustrating the existence of such fermionic isocurvature perturbations, we minimally extend the single field inflation by adding a stable massive fermion field coupled through a Yukawa coupling to a light noninflaton scalar field whose mass is much lighter than the fermion field

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[1] Even though the reheat degrees of freedom do not need to be dynamically important during the quasi-de Sitter (dS) era, multiple fields are certainly lurking in the scenario.

[2] Even with the massive fermions, we will be naturally concerned with light fermions where $m_\psi/H \ll 1$.

(hence, there are no decays of the scalars to the fermions). The light noninflaton scalar field (which is minimally coupled to gravity) serves as a conformal symmetry breaking sector through which the fermions will attain appreciable correlations. We compute the isocurvature two-point function of fermions that are gravitationally produced during inflation and identify the phenomenologically viable parameter space. We also estimate the local non-Gaussianity and show that it may be observationally large in a particular parametric regime.

At the technical level, treating fermionic isocurvature fluctuations during inflation requires composite operator renormalization in quasi-dS spacetime because the fermionic energy-momentum tensor is a composite bilinear operator (i.e. fermions cannot get vacuum expectation values) and the leading two-point function contribution involves a one-loop 1PI diagram. To our knowledge, this paper is the first paper to apply composite fermion operator renormalization in inflationary spacetime to treat isocurvature perturbations. Indeed, an improper treatment of the operator renormalization can in principle lead to answers that are many orders of magnitude off as we pointed out with bosonic composite operators [53]. We also show that a gravitational Ward identity plays an important role in suppressing the scalar metric perturbation interaction contribution to the isocurvature two-point function (thereby justifying our introduction of another scalar sector).

This paper is presented in the following order. In Sec. II, we motivate and discuss the fermion isocurvature model. Next, we review the definition of the gauge-invariant variables and the quantum operator associated with the cold dark matter (CDM) isocurvature in Sec. III. In Sec. III A, we present the regulator and the renormalization conditions for our isocurvature operator. We explain the constraints on the Yukawa coupling coming from the self-consistency of our simplified scenario in Sec. IV. In Sec. V, we compute the isocurvature 2-point function. The leading order and the next leading order results are given in Secs. V A and V B, and the power spectrum is presented in Sec. V C. In Sec. VI, we discuss the numerical implications of our results and non-Gaussianities. Afterwards in Sec. VII, we discuss the explicit computation of how a diffeomorphism Ward identity plays a role in suppressing the scalar metric perturbation contribution to the isocurvature two-point function. Finally, in Sec. VIII we summarize and conclude. Some technical details of the computations are given in the Appendixes.

II. FERMION ISOCURVATURE MODEL

As is well known, if any small mass fermion field degrees of freedom exist during inflation which is usually assumed to be a Bunch-Davies vacuum state, fermion particles will be produced gravitationally (see, e.g., [51,52,54]). The inhomogeneities of the gravitationally produced fermions will generically not align with the inhomogeneities of the inflaton, depending on its interactions. If most of the radiation in the universe comes from the inflaton decay, then the misalignment of the inhomogeneities of the fermions and the inflaton will lead to isocurvature perturbations [55–57].

Now, to motivate our fermion model with Yukawa interactions, it is important to understand why interactions to the conformal symmetry breaking sector is required. It is also well known that massless fermion classical action enjoys a conformal symmetry:

$$g_{\mu\nu} \to e^{2\sigma(x)} g_{\mu\nu}, \tag{1}$$

$$\psi \to e^{-3\sigma(x)/2}\psi. \tag{2}$$

Since FRW spacetime can be written as a conformal transformation of the Minkowski space [i.e. $a = \exp(\sigma)$], we would expect for a tree level fermion propagating on an FRW spacetime without any interactions with a conformal symmetry breaking sector

$$\langle \bar{\psi}\psi(t,\vec{x})\bar{\psi}\psi(t,\vec{y})\rangle_{\mathrm{conn}} = \langle \bar{\psi}_M\psi_M(t,\vec{x})\bar{\psi}_M\psi_M(t,\vec{y})\rangle_{\mathrm{conn}} a^{-6}, \tag{3}$$

where ψ_M is the Minkowski fermion. At leading order, there are no other scales in this function except $|\vec{x}-\vec{y}|$. Hence, we conclude

$$\langle \bar{\psi}\psi(t,\vec{x})\bar{\psi}\psi(t,\vec{y})\rangle_{\mathrm{conn}} \sim \frac{1}{a^6|\vec{x}-\vec{y}|^6} \tag{4}$$

in the massless limit.[3] We expect this to be the dominant contribution in the limit that $m_\psi/H \ll 1$. When $m_\psi/H \gg 1$, we also expect there can be factors multiplying this that vanish exponentially fast as $m_\psi/H \to \infty$ (we show this explicitly in Sec. V A). Hence, we expect Eq. (4) to be the leading order of magnitude composite correlator if the theory is approximately conformally invariant. As we will show below, the comoving gauge isocurvature perturbations are proportional to

$$\left\langle \frac{\delta\rho_\psi^{(C)}}{\bar{\rho}_\psi}\frac{\delta\rho_\psi^{(C)}}{\bar{\rho}_\psi}\right\rangle \sim \frac{\langle \bar{\psi}\psi(t,\vec{x})\bar{\psi}\psi(t,\vec{y})\rangle_{\mathrm{conn}}}{\langle\bar{\psi}\psi\rangle^2}, \tag{5}$$

where one sees the appearance of the suppressed correlator computed in Eq. (4). Because of this suppression, fermionic isocurvature perturbations require nontrivial interactions with a conformal symmetry breaking sector.

If the conformal symmetry breaking sector is just the ζ sector of the inflaton, then its effective coupling to the fermions is suppressed because there is an infinitesimal shift symmetry of the ζ coming from a residual

[3]The scaling behavior of the two-point correlator is similar to that of correlators considered in Ref. [58] in the context of conformal field theories.

diffeomorphism symmetry in the comoving gauge. (We will explain this explicitly in Sec. VII in terms of a Ward identity.) Hence, to generate an observable fermionic correlator during the horizon exit, another conformal symmetry breaking sector must be introduced which does not suffer from derivative coupling suppression similar to ζ.[4] We thus introduce a Yukawa coupling to a light noninflaton scalar and demonstrate that this interaction can induce observable isocurvature fluctuations.[5]

Given this motivation, let us now specify the model studied in this paper. We use one real scalar ϕ slow-roll inflaton degree of freedom that dominates the energy density during inflation and then perturbatively decays to the SM sector to reheat the universe. We also introduce another minimally coupled light real scalar degree of freedom σ which has no coupling to ϕ or the SM sector (necessary for reheating) stronger than gravity.[6] As we explained, the main role of σ is to provide a conformal symmetry breaking sector which can couple to the Dirac fermions ψ through a Yukawa coupling. We assume ψ is charged under a conserved discrete charge such that the one particle states are stable and can act as dark matter. Note that since we do not require all of the dark matter to come from the fermions, this system is consistent with the existence of the weakly interacting massive particle (WIMP) dark matter. Because ψ is too weakly interacting with the SM to be produced directly, gravitational production of ψ during and after inflation is significant and gives rise to nonthermal CDM and its isocurvature perturbations.

Such a model is described by the action[7]

$$S = \int (dx)\Bigg\{\mathcal{L}_{\rm inf}[g_{\mu\nu},\phi] + \mathcal{L}_{\rm SM+CDM}[g_{\mu\nu},\{\Psi\}] + \mathcal{L}_{\rm RH}[g_{\mu\nu},\phi,\{\Psi\}] + -\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{y}{4!}\sigma^4 + \bar{\psi}(i\gamma^a\nabla_{e_a} - m_\psi)\psi - \lambda\sigma\bar{\psi}\psi\Bigg\}, \quad (6)$$

where $M_p^2 = \frac{1}{8\pi G} = 1$, $(dx) \equiv \sqrt{-g}d^4x$, and $\mathcal{L}_{\rm inf}$ and $\mathcal{L}_{\rm SM+CDM}$ are the Lagrangians for the inflaton and the SM + CDM sectors, and $\mathcal{L}_{\rm RH}$ describes the sector responsible for reheating. Because an interesting parameter region exists for our scenario in which the ψ constitute a tiny fraction of the total dark matter content, the Lagrangian $\mathcal{L}_{\rm SM+CDM}$ describes the CDM sector different from ψ to make the scenario phenomenologically viable. Note that natural heavy dark matter candidates for ψ exist in the context of string phenomenology [59,60]. Furthermore, many extensions of the Standard Model also possess superheavy dark matter candidates (see, e.g., [61–71]). Since there are many scalar field degrees of freedom in typical beyond the standard models, the possibility of identifying one of these scalars with σ is also plausible. Although the cosmological phenomenology of weakly interacting dark matter on large scales has been investigated already in literature (see, e.g., [19,28,30,31,72,73]), our work is the first to describe fermionic fluctuation correlations during inflation. Note that although Eq. (6) has a quartic term σ^4, we will focus on the parametric region in which the quartic coupling y will be small and tuned against radiative generated quartic couplings from the Yukawa interaction to keep the effects of the σ interactions to a minimum. Hence, our effective parametric domain will be controlled by $\{\lambda, m_\sigma, m_\psi\}$.

We focus on a particular parametric region of $\{\lambda, m_\sigma, m_\psi\}$ such that σ only assists in generating large scale density perturbations of ψ, and the density perturbations and the relic abundance from the σ particles vanish or are suppressed compared to those from the ψ particles. For example, requiring that the correlator $\langle\sigma\sigma\rangle|_{t_*}$ relevant for the isocurvature perturbations not be suppressed gives the condition $m_\sigma/H(t_*) < 1$ where t_* is the time at which the fermion production ends. This implies $m_\sigma < m_\psi$ is the relevant parameter region. Furthermore, in order to prevent any large isocurvature perturbations and relic abundance of σ, we assume that the σ particles decay before σ becomes an important fluid component of the evolution of the universe (e.g., before matter-radiation equality). Note, however, that this restriction is a matter of simplicity. In general, we note that a weakly interacting and stable σ may also be phenomenologically allowed without problems regarding the relic abundance and the isocurvature from σ. Moreover, for simplicity, we restrict λ such that (1) $\sigma\sigma \to \bar{\psi}\psi$ via the Yukawa interactions is suppressed compared to the gravitational process in producing $\bar{\psi}\psi$, and (2) any $\sigma +$ gravity $\to \bar{\psi}\psi$ processes are estimated to be unimportant. This restriction is approximately equivalent to being in a parametric region where tree-level propagator neglecting resumption of the Yukawa interactions is valid.

In addition, in order to detach our model from the details of the inflationary model of ϕ, we focus on the light fermion ψ, such that $m_\psi < H_e$, where H_e is the Hubble scale at the end of inflation. This is because the gravitational particles production is generally sensitive to how the inflation ends in such a way that an extra suppression factor $\exp(-cm_\psi^2/H_e^2)$ (where c is a number depending on how

[4]Although we have not investigated the suppression for the tensor perturbation interactions with a full computation, we expect a similar suppression of the tensor perturbation interactions.

[5]Note that this introduction of a light scalar is not particularly attractive from a model building perspective since we provide no explicit mechanism to protect its light mass. We defer the challenge of building an attractive model to a future work since the purpose of this paper is to demonstrate the basic physics mechanism.

[6]For now, we will consider this as a tuning and will not address serious model building issues in this paper. It is plausible that this kind of scenario can be realized in the context of the supersymmetry hidden sector.

[7]Our metric convention is $(-,+,+,+)$.

the inflation connected with the post inflationary era) appears in the estimation of the gravitationally produced particle number density n_ψ. (Throughout the paper, we will sometimes distinguish H_e from $H_{\rm inf}$ which is defined to be the expansion rate during inflation.) On the other hand, if $m_\psi < H_e$, the factor becomes simply an $O(1)$ number, and particularly, for fermions we can estimate the number density $n_\psi(t_*)$ as $O(0.1)m_\psi^3$ at $H(t_*) \sim m_\psi$ regardless of how the inflation ends [51]. The physics of this universality is tied to the conformal symmetry of the fermions in the massless limit.

At this point, we emphasize that our model is different from other fermionic (e.g., gravitino) isocurvature models in literature (e.g., [49,74,75]). We explicitly predict the amplitudes of fermion density perturbations from a joint effect of the gravitational particle production and σ modulation on m_ψ via the matter loop diagrams. In contrast, in Refs. [49,74,75] the fermions are produced from the on-shell inflatons and/or curvatons (the latter has the closest identification in our model with σ) after the end of inflation. A sharp observable contrast of our model with these other models is that our scenario predicts an uncorrelated type of isocurvature (i.e. curvature-isocurvature cross-correlation is negligible) while these other models purportedly generate a correlated type of isocurvature. This is a consequence of the fact that these other models do not describe any fermionic fluctuations during inflation while in our model, the expansion during inflation imparts work to virtual fermionic fluctuations to put them on shell.

III. OPERATOR FOR ISOCURVATURE PERTURBATION

Recall that the scalar perturbation of the metric is parametrized as

$$\delta g_{\mu\nu}^{(S)} = \begin{pmatrix} -E & aF_{,i} \\ aF_{,i} & a^2[A\delta_{ij} + B_{,ij}] \end{pmatrix}. \qquad (7)$$

The gauge-invariant variables are constructed by combining metric perturbations and other perturbations, such as density perturbations. For example, the conventional first-order gauge-invariant perturbation associated with the energy density of a fluid a is defined (see, e.g., [76] and references therein) by

$$\zeta_a \equiv \frac{A}{2} - H\frac{\delta\rho_a}{\dot{\bar{\rho}}_a}. \qquad (8)$$

In particular, we define the conventional curvature perturbation as

$$\zeta \equiv \frac{A}{2} - H\frac{\delta\rho_{\rm tot}}{\dot{\bar{\rho}}_{\rm tot}}, \qquad (9)$$

where

$$\delta\rho_{\rm tot} = \sum_i \delta\rho_i, \qquad \bar{\rho}_{\rm tot} = \sum_i \bar{\rho}_i. \qquad (10)$$

This quantity ζ is conserved when modes are stretched out of the horizon even through the reheating era as long as it is set by the adiabatic initial condition, i.e., $\zeta = \zeta_a$ for any fluid a. Furthermore, if perturbations are generated solely by inflaton during inflation, such as the single field inflation, superhorizon perturbations automatically satisfy the adiabatic initial condition and the perturbations are conserved so that we can match them with those during the early radiation dominated (RD) era, $\zeta_\phi(t_{\rm inf}) = \zeta_\gamma(t_{\rm RD}) = \zeta_m(t_{\rm RD}) = \cdots$.

On the other hand, an isocurvature perturbation is defined by a relative density perturbation between two different fluids

$$\delta_{Sij} \equiv 3(\zeta_i - \zeta_j) = -3H\left(\frac{\delta\rho_i}{\dot{\bar{\rho}}_i} - \frac{\delta\rho_j}{\dot{\bar{\rho}}_j}\right). \qquad (11)$$

In general, it may arise during inflation if there are more than one degree of freedom. Although their mixing with perturbations of different fluids can lead to the failure of the conservation of the curvature perturbation ζ, such effects are negligible as for any species i whose $\bar{\rho}_i + \bar{P}_i$ is sufficiently smaller than $\bar{\rho}_{\rm tot} + \bar{P}_{\rm tot}$ until the universe reaches radiation domination. Particularly, for gravitationally produced fermions, we have

$$\left.\frac{\bar{\rho}_\psi + \bar{P}_\psi}{\bar{\rho}_{\rm tot} + \bar{P}_{\rm tot}}\right|_{t_*} \sim \left.\frac{\bar{\rho}_\psi}{\bar{\rho}_{\rm tot}}\right|_{t_*} \sim \frac{m_\psi^2}{M_p^2} \ll \Delta_\zeta^2, \qquad (12)$$

where t_* is the time that the gravitational fermion production ends, $H(t_*) \sim m_\psi$. Hence, we expect the superhorizon curvature perturbation to be approximately conserved through the reheating, $\zeta(t_{\rm RD}) \approx \zeta_\phi(t_{\rm inf})$.

The dominant fraction of the produced fermions are nonrelativistic.[8] Then the fermion energy density behaves as[9]

$$\frac{d}{dt}\bar{\rho}_\psi(t) \approx -3H\bar{\rho}_\psi \quad \text{for} \quad t > t_*, \qquad (13)$$

and from Eq. (11) a general CDM isocurvature is written as

[8]This is a valid assumption because gravitationally excited fermion modes that contribute to the energy density are less than the fermion mass, i.e., $|\beta_k|^2$ for $k/a \lesssim m_\psi$, where β_k is the Bogoliubov coefficient. See Appendix B for details.

[9]One can find that $\bar{\rho}_\psi \propto a^{-3}(t)$ for $t > t_*$ if $\bar{\rho}_\psi$ is renormalized by the adiabatic subtraction. See Appendix B and Ref. [54]. Then we can treat ψ as a pressure less matter.

$$\delta_S = \frac{\delta\rho_{\rm CDM}}{\bar{\rho}_{\rm CDM}} - \frac{3}{4}\frac{\delta\rho_\gamma}{\bar{\rho}_\gamma}. \tag{14}$$

As discussed in Sec. II, the CDM may include decay products of the inflaton ϕ. Thus the CDM density perturbation is generally expressed as

$$\frac{\delta\rho_{\rm CDM}}{\bar{\rho}_{\rm CDM}} = \omega_\psi \frac{\delta\rho_\psi}{\bar{\rho}_\psi} + (1-\omega_\psi)\frac{\delta\rho_m}{\bar{\rho}_m}, \tag{15}$$

where the subscript m denotes the CDM component associated with the inflaton decay products (such as WIMPs of minimal supersymmetric models), and

$$\omega_\psi \equiv \bar{\rho}_\psi/(\bar{\rho}_\psi + \bar{\rho}_m). \tag{16}$$

In particular, in the comoving gauge ($\delta\rho_\phi/\dot{\bar{\rho}}_\phi = \delta\rho_m/\dot{\bar{\rho}}_m = \delta\rho_\gamma/\dot{\bar{\rho}}_\gamma = 0$), the CDM isocurvature becomes

$$\delta_S^{(C)} \approx \omega_\psi \frac{\delta\rho_\psi^{(C)}}{\bar{\rho}_\psi}, \tag{17}$$

where the superscript denotes the gauge choice.

Under the nonrelativistic assumption, we also approximate the fermion mass term $m_\psi\bar{\psi}\psi$ as its energy density[10]

$$\rho_\psi \approx m_\psi\bar{\psi}\psi, \tag{18}$$

and then the fermion isocurvature perturbation becomes

$$\delta_S^{(C)} \approx \omega_\psi \frac{\rho_\psi - \langle\rho_\psi\rangle}{\langle\rho_\psi\rangle} = \omega_\psi \frac{\bar{\psi}\psi - \langle\bar{\psi}\psi\rangle}{\langle\bar{\psi}\psi\rangle}. \tag{19}$$

Notice that as it is a quantum composite operator, we renormalize it with regulators and counterterms invariant under the underlying gauge symmetry, diffeomorphism in this case. In the following subsection, we present the technical detail of the composite operator renormalization. From now on, we will use the comoving gauge in calculating the correlation function and drop the superscript (C) for convenience.

[10] Using the adiabatic vacuum prescription, the renormalized energy density is approximated in the nonrelativistic case as

$$\langle(\rho_\psi)_r\rangle \approx m_\psi\langle N_\psi\rangle = 2m_\psi \int \frac{d^3k}{(2\pi^3)}\frac{1}{a^3}|\beta_k|^2,$$

where N_ψ is a fermion number operator, and the subscript r denotes that the operator is a renormalized composite operator. This quantity is in accord with

$$m_\psi\langle(\bar{\psi}\psi)_r\rangle = 2m_\psi \int \frac{d^3k}{(2\pi^3)}\frac{m_\psi}{\omega_p}|\beta_k|^2 \approx 2m_\psi \int \frac{d^3k}{(2\pi^3)}|\beta_k|^2.$$

In particular, $(\bar{\psi}\psi)$ has an advantage in constructing gauge-invariant variables because it is manifestly 4-scalar, but N_ψ.

A. Regularization and renormalization for isocurvature perturbation

In this subsection, we explain our regularization procedure and renormalization scheme that determines the counterterms. The most crucial renormalization condition that the isocurvature perturbations are sensitive to is Eq. (38).

For the convenience of preserving covariance and incorporating the adiabatic vacuum boundary condition, we use Pauli-Villars (PV) regularization [77]. This involves the replacements

$$\psi \to \psi + \sum_n \psi_n, \qquad \sigma \to \sigma + \sum_n \sigma_n, \tag{20}$$

and the addition of the Pauli-Villars part in the free Lagrangian

$$\mathcal{L}_{\rm PV} = \sum_{n=1} C_n\left(-\frac{1}{2}g^{\mu\nu}\partial_\nu\sigma_n\partial_\nu\sigma_n - \frac{1}{2}M_n^2\sigma_n^2\right) \tag{21}$$

$$+ \sum_{n=1} D_n\bar{\psi}_n(i\gamma^a\nabla_a - m_n)\psi_n. \tag{22}$$

For notational simplicity, we let $C_0 = 1$, $M_0 = m_\sigma$ and $D_0 = 1$, $m_0 = m_\psi$, and let index $N = 0, 1, \ldots$, and $n = 1, 2, \ldots$. We require the following constraints for scalar regulators:

$$\sum_N C_N^{-1} = 0, \quad \sum_N C_N^{-1}M_N^2 = 0, \quad \sum_N C_N^{-1}M_N^4 = 0, \ldots \tag{23}$$

and the following constraints for fermion regulators:

$$\sum_N D_N^{-1} = 0, \quad \sum_N D_N^{-1}m_N = 0, \quad \sum_N D_N^{-1}m_N^2 = 0, \ldots, \tag{24}$$

where we need to introduce sufficient numbers of PV fields and constraints to cancel all the divergences. Notice the additional constraints in the fermions with odd powers of m_N.

With the operator dimension and the symmetry considered, the renormalized operator is written as

$$(\bar{\psi}\psi)_{x,r} = (\bar{\psi}_x)_r(\psi_x)_r(1+\delta Z_1) + \delta Z_2(\sigma_{x,r})^3 + \delta Z_3(\sigma_{x,r})^2 + \delta Z_4\sigma_{x,r} + \delta Z_5 + \delta Z_6\Box\sigma_{x,r} + \delta Z_7 R + \delta Z_8 R\sigma_{x,r}, \tag{25}$$

where each field operator should be understood as including a sum of the PV fields as in Eq. (20). Then we give the renormalization conditions to determine the counterterms. For δZ_i which are not coupled to R, $R_{\mu\nu}$, $R^\alpha_{\beta\mu\nu}$, and their derivatives, we can go to the Minkowski space and impose the renormalization conditions there. (Of course, we do not need to separate the curved space contribution and the flat space contribution with two computations, but we present it here this way here for clarity in the physical partition.) We define the renormalized operator $\bar{\psi}\psi$ at one-loop order, such that it measures the number density of the fermion particles. First, we require its expectation value in the flat space vacuum to vanish:

$$\langle \mathrm{vac}|\bar{\psi}\psi(x)|\mathrm{vac}\rangle_{\mathrm{flat}} + \sum_{n=1}\langle \mathrm{vac}|\bar{\psi}_n\psi_n(x)|\mathrm{vac}\rangle_{\mathrm{flat}} + \delta Z_5 = 0 \tag{26}$$

$$\Rightarrow -\int \frac{d^4p}{(2\pi)^4}\sum_N D_N^{-1}\mathrm{Tr}\left\{\frac{1}{i}\frac{-\not{p}+m_N}{p^2+m_N^2-i\epsilon}\right\} + \delta Z_5 = 0. \tag{27}$$

This corresponds to the evaluation of Fig. 1(a).

Next, we impose the renormalization condition consistent with the fact that as far as the fermion sector is concerned, a shift of σ by a constant in the tree-level action is equivalent to a shift in the mass of the fermion. More explicitly, we demand that if σ is shifted as $\sigma \to \sigma + c$, the one-point function satisfies

$$\langle \mathrm{vac}|(\bar{\psi}\psi)_{x,r}|\mathrm{vac}\rangle_{\mathrm{flat}} = \langle \mathrm{vac}|[(\bar{\psi}\psi)_{x,r} + \Delta(\bar{\psi}\psi)_{x,r}]|\mathrm{vac}\rangle_{\mathrm{flat}},$$
$$\mathcal{L}_I = -\lambda c \bar{\psi}_y\psi_y, \tag{28}$$

where $\Delta(\bar{\psi}\psi)_{x,r}$ corresponds to a shift in the σ dependent composite operator counterterms and $\mathcal{L}_I$ corresponds to the c dependent mass shift Lagrangian term. This leads to the diagrams in Figs. 1(c)–1(e) with the external σ propagators truncated and fixes δZ_2, δZ_3, δZ_4:

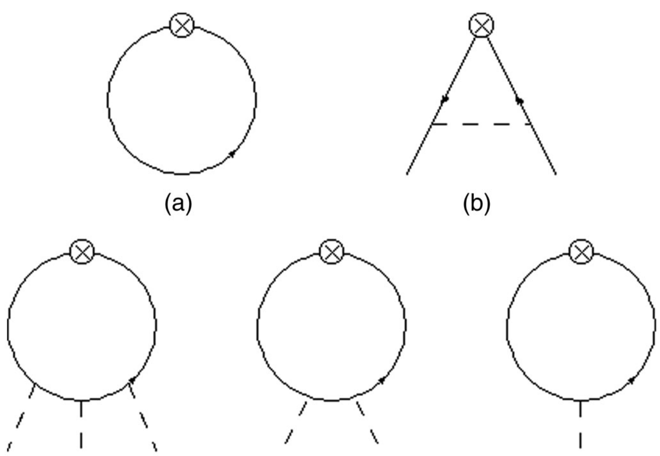

FIG. 1. Diagrams determining the counter-terms where the solid line corresponds to the fermion line and the dashed lines corresponds to σ lines. Diagram (a) determines δZ_5 and δZ_7, and Diagram (e) does δZ_4, δZ_6 and δZ_8. Diagrams (b), (c), and (d) fix δZ_1, δZ_2, and δZ_3, respectively. It is convenient to truncate the external σ legs on diagrams (c), (d), and (e) with zero momentum insertion, making these mass insertions.

$$-(-i\lambda)^3\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left\{\left(\sum_M D_M^{-1}\frac{1}{i}\frac{-\not{k}+m_M}{k^2+m_M^2-i\epsilon}\right)^4\right\} + \delta Z_2 = 0, \tag{29}$$

$$-(-i\lambda)^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left\{\left(\sum_M D_M^{-1}\frac{1}{i}\frac{-\not{k}+m_M}{k^2+m_M^2-i\epsilon}\right)^3\right\} + \delta Z_3 = 0, \tag{30}$$

and

$$-i\lambda\int d^4y\langle(\bar{\psi}\psi)_x(\bar{\psi}\psi)_y\rangle + \delta Z_4 = 0 \tag{31}$$

$$\Rightarrow -i\lambda\int\frac{d^4k}{(2\pi)^4}(-)\mathrm{Tr}\left\{\left(\sum_M D_M^{-1}\frac{1}{i}\frac{-\not{k}+m_M}{k^2+m_M^2-i\epsilon}\right)^2\right\} + \delta Z_4 = 0. \tag{32}$$

Furthermore, we require $\bar{\psi}\psi$ to have no loop corrections when contracted with on-shell fermion. This leads to Fig. 1(b) (where we have set the composite operator momentum to be 0 for convenience) and fixes δZ_1:

$$\delta Z_1 + (i\lambda)^2\int\frac{d^4k}{(2\pi)^4}\sum_{L,M,N}C_L^{-1}D_M^{-1}D_N^{-1}\frac{1}{i}\frac{1}{k^2+M_L^2-i\epsilon} \times \frac{1}{i}\frac{[-k-\not{p}+m_M]}{(k+p)^2+m_M^2-i\epsilon}\times\frac{1}{i}\frac{[-k-\not{p}+m_N]}{(k+p)^2+m_N^2-i\epsilon} = 0. \tag{33}$$

Similarly, we demand $\bar{\psi}\psi$ to have no loop corrections when contracted with on-shell scalar line. Explicitly, the diagram corresponds to Fig. 1(e) determining δZ_6:

$$-i\lambda\int d^4y\langle(\bar{\psi}\psi)_x(\bar{\psi}\psi)_y\rangle e^{ip\cdot y} + \delta Z_4 - p^2\delta Z_6 = 0 \tag{34}$$

$$\Rightarrow i\lambda\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left\{\sum_M D_M^{-1}\frac{1}{i}\frac{-\not{k}+m_M}{k^2+m_M^2-i\epsilon} \times\sum_N D_N^{-1}\frac{1}{i}\frac{-\not{k}-\not{p}+m_N}{(k+p)^2+m_N^2-i\epsilon}\right\} + \delta Z_4 - p^2\delta Z_6 = 0, \tag{35}$$

where $p^2 = -m_\sigma^2$.

For δZ_i that depend on curved spacetime nature, we match the renormalized result to that from the adiabatic subtraction. In order to fix δZ_7, we impose the number density $\langle in|(\bar{\psi}\psi)_{r,x}|in\rangle$ to be the density defined by the adiabatic prescription (see, e.g., [29,51,52,54,78–80]):

$$n_\psi \equiv \langle in|\bar{\psi}\psi(x)|in\rangle + \sum_{n=1}\langle in|\bar{\psi}_n\psi(x)_n|in\rangle + \delta Z_5 + \delta Z_7 R(x) \tag{36}$$

$$= \langle in|\bar{\psi}\psi(x)|in\rangle - \langle \mathrm{WKB, vac}, t_x|\bar{\psi}\psi(x)|\mathrm{WKB, vac} t_x\rangle, \tag{37}$$

where $|\mathrm{WKB, vac}, t_x\rangle$ is the WKB vacuum defined at t_x by the adiabatic prescription. The diagram of interest is shown in Fig. 1(a), and the divergent part of δZ_7 determined this way is linear in the fermion mass.

In order to determine δZ_8, we repeat the consideration analogous to Eq. (32) on a background field $\sigma(x) = c$, where c is an infinitesimal constant. Since a constant σ shift is equivalent to a shift of the fermion mass, we want to choose δZ_8 to get

$$\lambda\partial_m n_\psi(x) = -i\lambda\int_{\mathrm{CTP}}(dy)\sum_{N,M}\langle in|P\{\bar{\psi}_M(x)\psi_N(x)\bar{\psi}_N(y)\psi_M(y)\}|in\rangle_{\mathrm{conn}} + \delta Z_4 + \delta Z_8 R(x), \tag{38}$$

where the subscript CTP denotes closed-time path, and P is the path-ordering operator for an "in-in" exception value. (For example, see Refs. [81,82].) Note that the diagram of interest corresponds to Fig. 1(e). As we will see later, this renormalization condition plays a crucial role in determining the isocurvature correlator. The solution for all the δZ_i can be expressed in terms of Feynman parameter integrals. However, such explicit expressions are not relevant to determine the isocurvature correlation function. In contrast the left hand side of Eq. (38) is important.

To summarize, we have given a prescription to regularize and renormalize the composite operator $\bar{\psi}\psi$. The renormalization conditions ensure that $\langle in|(\bar{\psi}\psi)_{r,x}|in\rangle$ agrees with that defined by the adiabatic prescription in curved spacetime, and they also ensure that a constant shift in σ is equivalent to a constant shift in the fermion mass. Note that because the gravitational production of fermions is still in flux when $m_\psi < H$, we evaluate the number density n_ψ later than t_*, where $H(t_*) \sim m_\psi$, as far as the renormalization conditions are concerned.

IV. SCENARIO CONSTRAINTS ON SCALAR FIELD σ

In this section, we explain the constraints on the Yukawa coupling λ that comes from requiring σ to behave as an unscreened long range force carrier whose on-shell particle states do not significantly participate in ψ production.

We will find that the $\langle\sigma\sigma\rangle|_{t_*}$ power spectrum relevant for the isocurvature perturbations is not suppressed if $m_\sigma/H(t_*) < 1$ where t_* is the time at which $H(t_*) = m_\psi$ (i.e. t_* is the time at which the fermion + antifermion number freezes [51]). This implies $m_\sigma < m_\psi$ is the relevant parameter region for the scenario of this paper. Furthermore, in order to prevent any large isocurvature perturbations and a relic abundance of σ, we assume that $\langle\sigma\rangle = 0$ and the σ particles decay before σ becomes an important fluid component of the evolution of the universe (e.g., before matter-radiation equality). Note, however, that this restriction is a matter of simplicity. There exist parameter regions in (m_σ, λ) such that σ survives as a long-lived weakly interacting particle (i.e. a dark matter). However, in such cases, the constraints from the relic abundance and the isocurvature of σ restrict the σ mass to be very small, e.g., $m_\sigma \lesssim 10^{-6}$ eV for $H_{\mathrm{inf}} \sim 10^{13}$ GeV. (See, e.g., [72,83–85] for the parametric bounds for the QCD axion produced by inflation.) In principle, it is possible to build a model that has such small m_σ with the help of some underlying symmetry, such as a shift symmetry.

Although we assume that $m_\sigma < m_\psi$, σ would generally acquire a plasma mass correction through interactions with an ensemble of ψ particles. Thus we consider the effect of the produced ψ on the σ correlator and show that the effect is negligible. We expect the fermions do not affect scalar modes before the horizon exit because the mass correction by the fermion is still small compared to the Hubble friction during inflation. After the scalar mode exits the horizon, the fermions exert a tiny computable drag on σ. The equation of motion of σ from the action (6)[11] is written as

$$0 = \langle in|[(\Box - m_\sigma^2)\sigma_x - \lambda\bar{\psi}\psi_x + \delta Z_0 + \delta Z_R R_x + \delta Z_\sigma\Box\sigma_x - \delta m_\sigma^2\sigma_x + \delta Z_\xi R_x\sigma_x][\cdots]|in\rangle \tag{39}$$

$$= (\Box_x - m_\sigma^2)\langle\sigma_x[\cdots]\rangle + i\lambda^2\int^x(dz)\langle[\bar{\psi}\psi_x, \bar{\psi}\psi_z]\rangle\langle\sigma_z[\cdots]\rangle + (\delta Z_\sigma\Box_x - \delta m_\sigma^2 + \delta Z_\xi R_x)\langle\sigma_x[\cdots]\rangle + (\delta Z_0 + \delta Z_R R_x - \lambda\langle\bar{\psi}\psi_x\rangle)\langle[\cdots]\rangle + O(\lambda^3, y), \tag{40}$$

where $[\cdots]$ denotes any quantum operators in the correlation function. We choose the counterterms δZ_0 and δZ_R such that the tadpole $\langle\sigma\rangle$ vanishes, i.e., $(\delta Z_0 + \delta Z_R R - \lambda\langle\bar{\psi}\psi\rangle) = 0$, where the PV regulator is assumed. Moreover, when σ varies very slowly outside the horizon, we factor $\langle\sigma_z[\cdots]\rangle$ out of the integral in Eq. (40), and we renormalize the integral using the counterterms $(\delta Z_\sigma\Box_x - \delta m_\sigma^2 + \delta Z_\xi R_x)\langle\sigma_x[\cdots]\rangle$ such that

[11]The counterterms appearing in the action includes

$$S_{\mathrm{c.t.}} \ni \int(dx)\left[-\frac{1}{2}\delta Z_\sigma(\partial\sigma)^2 - \frac{1}{2}\delta m_\sigma^2\sigma^2 + \delta Z_0\sigma + \delta Z_R R\sigma + \delta Z_\xi R\sigma^2\right].$$

Note that the linear σ terms exist in the action because the action does not preserve the Z_2 symmetry due to the Yukawa coupling.

the result is consistent with that obtained by the adiabatic subtraction[12]:

$$i\lambda^2 \int^x (dz)\langle[\bar{\psi}\psi_x, \bar{\psi}\psi_z]\rangle + (-\delta m_\sigma^2 + \delta Z_\xi R_x) = -\lambda^2 \left(\frac{\partial n_\psi}{\partial m_\psi}\right), \tag{41}$$

where n_ψ is the renormalized fermion number density defined by Eq. (37), and we have used Eq. (38) in the derivation. Therefore, we find the effective mass of σ when it slowly varies (i.e., $k/a \ll H$ and $m_\sigma \ll H$)

$$m_\sigma^{\rm eff} = m_\sigma^2 + \Delta m_\sigma^2(t) \approx m_\sigma^2 + \lambda^2 \frac{\partial n_\psi(t)}{\partial m_\psi}. \tag{42}$$

Because we estimate $n_\psi \lesssim O(0.1)(m_\psi H)^{3/2}$ when $m_\psi \lesssim H$,[13] based on dimensional analysis, we expect that the mass correction by the ψ loop is

$$\Delta m_\sigma^2(t) \approx \lambda^2 \frac{\partial n_\psi(t)}{\partial m_\psi} \sim \begin{cases} O(0.1 \text{ or less})\lambda^2 m_\psi^{1/2} H^{3/2} & \text{for} \quad m_\psi < H(t) \\ O(0.1)\lambda^2 m_\psi^2 & \text{for} \quad m_\psi > H(t) \end{cases}. \tag{43}$$

Therefore, in general, before the fermion production ends $m_\psi < H$, this scalar mass correction Δm_σ^2 does not ruin the stability of our scenario $m_\sigma^2 + \Delta m_\sigma^2(t) < m_\psi^2 < H^2(t)$ as long as $m_\sigma^2 < m_\psi^2$.

Next, we ask the question of which parametric region would be consistent with the simplifying assumption that ψ particles are primarily produced gravitationally and not by σ. To this end, we first consider the annihilation $\sigma\sigma \to \bar{\psi}\psi$. The annihilation is the most significant at the end of inflation because ψ particles produced from σ before the end of inflation are diluted, and $\sigma\sigma \to \bar{\psi}\psi$ after the end of inflation is also limited because the allowed kinematic phase space is redshifted. Thus we compare the number density of the produced ψ from σ at the end of inflation, $n_{\sigma\to\psi}$ with that of gravitationally produced ψ, $n_\psi(t_*) \sim m_\psi^3$, and we require their ratio to be small:

$$\left(\frac{a_e}{a(t_*)}\right)^3 \frac{n_{\sigma\to\psi}(t_e)}{n_\psi(t_*)} \sim \left(\frac{a_e}{a(t_*)}\right)^3 \frac{n_\sigma \Gamma(\sigma\sigma\to\psi\psi)\Delta t|_{t_e}}{n_\psi(t_*)} \tag{44}$$

$$\sim \left(\frac{H(t_*)}{H_e}\right)^2 \frac{H_e^3 \cdot \frac{\lambda^4}{16\pi^2} H_e \cdot \frac{1}{H_e}}{H^3(t_*)}$$

$$\sim \frac{\lambda^4}{16\pi^2} \frac{H_e}{m_\psi} \lesssim 1, \tag{45}$$

where the subscript e means a variable is evaluated at the end of inflation t_e.

Even though $m_\sigma < m_\psi$, the decay production of ψ through $\sigma \to \bar{\psi}\psi$ may still be possible if σ is sufficiently off shell due to its interactions with finite density of ψ in the subhorizon region [the subhorizon physics here is different from the superhorizon physics considered in Eq. (42)]. To turn off this channel, we require that the σ mass corrections from the fermion number density at the time of the end of inflation be small. This requires

$$\lambda^\kappa \frac{H_e/(2\pi)}{m_\psi} \lesssim 1, \tag{46}$$

where $\kappa \gtrsim O(1)$. To see how $\kappa \gtrsim O(1)$ can come about, consider the following estimate of the subhorizon thermal effect. The maximum effective number density of fermions at the end of inflation is

$$n_\psi(t_e) \lesssim 4m_\psi \left(\frac{H_e}{2\pi}\right)^2. \tag{47}$$

The energy density associated with these fermions is

$$\Delta V \sim n_\psi(t_e)\sqrt{\left(\frac{H_e}{2\pi}\right)^2 + \lambda^2\sigma^2}, \tag{48}$$

where we neglected $m_\sigma \ll H_e/(2\pi)$. This leads to an effective m_σ correction of

$$\Delta m_\sigma^2 \sim n_\psi(t_e)\frac{\lambda^2}{H_e/(2\pi)} \lesssim 4\lambda^2 m_\psi \left(\frac{H_e}{2\pi}\right). \tag{49}$$

Kinematically blocking the σ decay into ψ, we find

$$4\lambda^2 \left(\frac{H_e}{2\pi}\right) < m_\psi, \tag{50}$$

which corresponds to $\kappa = 2$. Note that this condition is more restrictive than Eq. (45).

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[12]In other words, we identify $-\delta m_\sigma^2$ and δZ_ξ with δZ_4 and δZ_8 in Eq. (38), and $\delta Z_\sigma \Box$ is neglected since σ is slowly varying.

[13]Note that the adiabatic prescription to determine the number density n_ψ does not apply for modes $m_\psi < k/a < \sqrt{m_\psi H}$ when $m_\psi < H$ because vacuum varies nonadiabatically, i.e., the adiabaticity parameter $\epsilon_k \equiv \frac{m_\psi k_p H}{\omega_k} \gtrsim 1$, where $k_p = k/a$ and $\omega_k = \sqrt{k_p^2 + m_\psi^2}$. See Appendix B for details. However, we can estimate the upper bound of the number density as

$$n_\psi(t) = \int \frac{d^3k_p}{(2\pi)^3}|\beta_k|^2 \lesssim \int^{\sqrt{m_\psi H}} \frac{d^3k_p}{(2\pi)^3}\frac{1}{2} \sim O(0.1)(m_\psi H)^{3/2} \quad \text{for} \quad t < t_*.$$

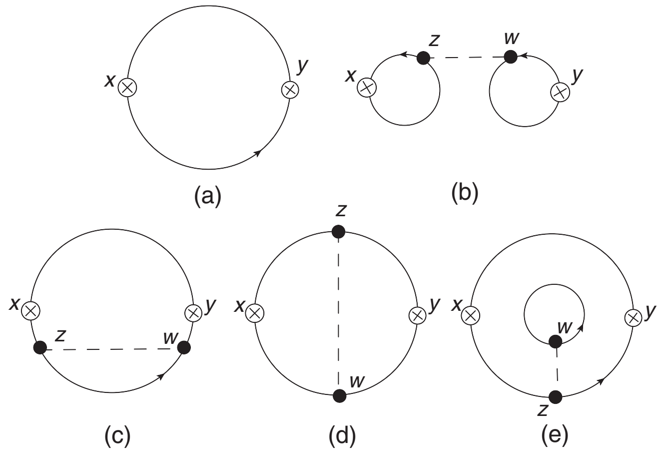

FIG. 2. The leading order diagram (a) and the next leading order diagrams (b), (c), (d), and (e) contributing to $\langle\bar{\psi}\psi_x\bar{\psi}\psi_y\rangle$, where the cross-dot vertices corresponds to $\bar{\psi}\psi$ insertion. Comparing the large r ($r \equiv |\vec{x}-\vec{y}|$) behavior of the equal-time correlator of the fermion and the scalar field, we show that diagram (b) dominates in the limit $r \to \infty$.

In sum, requiring σ to behave as an unscreened long range force carrier whose on-shell particle states do not significantly participate in ψ production gives a constraint on λ. The strongest condition is given by Eq. (46) with $\kappa \gtrsim O(1)$.

V. ISOCURVATURE TWO-POINT FUNCTION

In this section, we evaluate the two-point function of the renormalized isocurvature operator δ_S, given by Eq. (19). The average number density was computed in [51], the result is summarized in Appendix B. We only need to evaluate $\langle(\bar{\psi}\psi)_{x,r}(\bar{\psi}\psi)_{y,r}\rangle_c$. Since we want to use the quantum computation to set the initial condition for the subsequent classical fluid evolution, we will choose the time of the evaluation such that both the quantum and the classical fluid descriptions apply. We take $x^0 = y^0 = \eta_f$ at a time after the particle production ends, since the fluid description cannot describe the particle production process. We will take the separation $|\vec{x}-\vec{y}|$ to be large enough such that the intersection of their past light cone $I^-(x) \cap I^-(y)$ lives deep within the inflationary era. This ensures that the contributions from late-time short distance physics (e.g., reheating, phase transition) are minimized. The relevant diagrams for $\langle(\bar{\psi}\psi)_{x,r}(\bar{\psi}\psi)_{y,r}\rangle_c$ are given in Fig. 2. The crossed dot represents $(\bar{\psi}\psi)_{x,r}$ insertion, the solid dot represents the Yukawa interaction vertex, the dashed line represents the scalar σ propagator, and the solid line represents the fermion propagator.

A. Leading order result

We first consider the leading order diagram in Fig. 2(a). The diagram is explicitly written as

$$\langle\bar{\psi}\psi_x\bar{\psi}\psi_y\rangle_{(a)} = -\mathrm{Tr}[\langle\psi_x\bar{\psi}_y\rangle\langle\psi_y\bar{\psi}_x\rangle] = \sum_{i,j}\bar{V}_{i,x}U_{j,x}\bar{U}_{j,y}V_{i,y}. \quad (51)$$

Using a contour integration technique, we can evaluate the mode-sum analytically. The details are given in Appendix C. The result[14] is

$$\langle\bar{\psi}\psi_x\bar{\psi}\psi_y\rangle_{\mathrm{LO}} = \begin{cases} \frac{1}{\pi^4 a_x^6|\vec{x}-\vec{y}|^6}\left(1+O[(\frac{m_\psi}{H_{\mathrm{inf}}})^2]\right) & (m_\psi \ll H_{\mathrm{inf}}) \\ \frac{1}{\pi^4 a_x^6|\vec{x}-\vec{y}|^6}(4\pi)(\frac{m_\psi}{H_{\mathrm{inf}}})^3\exp(-2\pi\frac{m_\psi}{H_{\mathrm{inf}}}) & (m_\psi \gg H_{\mathrm{inf}}) \end{cases}, \quad (52)$$

where H_{inf} is the expansion rate during inflation. We can understand this result by backtracking the two points x, y to the time when they were deep inside the horizon and seeing what happened as they grew apart.

In the heavy mass case ($m_\psi \gg H_{\mathrm{inf}}$), the Compton radius m_ψ^{-1} is smaller than the horizon radius H_{inf}^{-1}. The physical separation r_{phys} will first grow to the Compton wavelength and trigger the exponential suppression factor $\exp(-2m_\psi r_{\mathrm{phys}})$ in the correlator,

$$\langle\bar{\psi}\psi_x\bar{\psi}\psi_y\rangle_{\mathrm{flat},m_\psi r_{\mathrm{phys}}>1} \sim \frac{m_\psi^3}{4\pi^3 r_{\mathrm{phys}}^3}\exp(-2m_\psi r_{\mathrm{phys}}). \quad (53)$$

As the physical separation r_{phys} grows further to exceed the horizon radius H_{inf}^{-1}, the correlator would freeze and start decreasing as $(a_r/a_\eta)^6$, where $a_r = 1/(H_{\mathrm{inf}}r)$ denote the scale factor at the horizon crossing. Substituting $a_r = \frac{1}{H_{\mathrm{inf}}r}$ and $r_{\mathrm{phys}} = H_{\mathrm{inf}}^{-1}$, we recover the heavy mass formula:

$$\left(\frac{a_r}{a_\eta}\right)^6 \frac{m_\psi^3}{4\pi^3 r_{\mathrm{phys}}^3}\exp(-2m_\psi r_{\mathrm{phys}}) \sim \frac{1}{a_x^6 r^6}\left(\frac{m_\psi}{H_{\mathrm{inf}}}\right)^3 \exp\left(-2\frac{m_\psi}{H_{\mathrm{inf}}}\right). \quad (54)$$

In the light mass case ($m_\psi \ll H_{\mathrm{inf}}$), the physical distance will cross the horizon radius first, without the exponential suppression of $\exp(-2m_\psi r_{\mathrm{phys}})$. From the flat space UV limit result $\frac{1}{r_{\mathrm{phys}}^6}$,

[14]Note that we do not consider the heavy mass case, $m_\psi \gg H_{\mathrm{inf}}$ where H_{inf} is the expansion rate during inflation, for the isocurvature because the estimation of the particle production depends on how the inflation ends as described in Sec. II. However, we provide the leading order of the two-point function to develop better intuition for the behavior of super horizon modes of ψ.

$$\langle\bar{\psi}\psi_x\bar{\psi}\psi_y\rangle_{\mathrm{flat},mr_{\mathrm{phys}}<1}\sim\frac{1}{r_{\mathrm{phys}}^6},\tag{55}$$

we use $a_r=\frac{1}{H_{\mathrm{inf}}r}$ and $r_{\mathrm{phys}}=H_{\mathrm{inf}}^{-1}$ to obtain

$$\left(\frac{a_r}{a_\eta}\right)^6\frac{1}{r_{\mathrm{phys}}^6}\sim\frac{1}{a_x^6r^6}.\tag{56}$$

Thus we recover the light mass result.

Unfortunately, the fractional relic density fluctuation at the CMB scale[15] is too small

$$\frac{\langle\delta\rho_x\delta\rho_y\rangle}{\langle\bar{\rho}_\psi\rangle^2}\sim\frac{m_\psi^2/(\pi^4a^6r_{\mathrm{CMB}}^6)}{m_\psi^2m_\psi^6(a_*^6/a^6)}\sim\left(\frac{1}{a_*m_\psi r_{\mathrm{CMB}}}\right)^6,\tag{57}$$

where r_{CMB} is the comoving distance for the typical CMB observation scale and the subscript $*$ denotes the time when fermion production ends. Let a_{CMB} denote the scale factor when the CMB scale exits the horizon, and then we have

$$r_{\mathrm{CMB}}^{-1}\sim a_{\mathrm{CMB}}H_{\mathrm{inf}}.\tag{58}$$

Assuming the fermion production ends during reheating when $m_\psi=H(t_*)$, and $H\propto a^{-\alpha}$ during reheating, then we have

$$\frac{a_eH_{\mathrm{inf}}}{a_*m_\psi}\sim\frac{a_eH_e}{a_*H_*}\sim\left(\frac{a_e}{a_*}\right)^{1-\alpha}\sim\left(\frac{H_e}{H_*}\right)^{1-\frac{1}{\alpha}}.\tag{59}$$

Assuming that inflation ends 50 e-folds after the CMB scale exits the horizon and a reheating effectively like a matter dominated period, i.e., $\alpha=3/2$, then we have

$$\frac{\langle\delta\rho_x\delta\rho_y\rangle}{\langle\bar{\rho}_\psi\rangle^2}\sim\left(\frac{a_{\mathrm{CMB}}H_{\mathrm{inf}}}{a_*m_\psi}\right)^6\sim\left(\frac{a_{\mathrm{CMB}}}{a_e}\frac{a_eH_{\mathrm{inf}}}{a_*m_\psi}\right)^6\sim e^{-300}\left(\frac{H_e}{m_\psi}\right)^2.\tag{60}$$

Using the fermion relic abundance formula (for $T_{\mathrm{RH}}=10^9$ GeV and $g_*=100$ cases) $\omega_\psi\sim(m_\psi/10^{10}\ \mathrm{GeV})^2$, we obtain

$$\frac{\langle\delta\rho_x\delta\rho_y\rangle}{\rho_{\mathrm{tot}}^2}\sim\omega_\psi^2\frac{\langle\delta\rho_x\delta\rho_y\rangle}{\langle\bar{\rho}_\psi\rangle^2}\sim e^{-300}\left(\frac{H_e}{10^{10}\ \mathrm{GeV}}\right)^2.\tag{61}$$

We thus find that generically the pure fermion isocurvature is very small on scales relevant for the CMB.

B. Next leading order result

We consider the diagrams in Figs. 2(b)–2(e), which contain the effects of the Yukawa interaction to the fermion production. We can perturbatively compute the diagrams using the "in-in" formalism (e.g., see Refs. [86,87] and references therein).

First, we estimate which diagram gives the largest contribution when x and y have large spatial separations. From the fact that equal-time correlator $\langle\sigma_x\sigma_y\rangle$ scales as $r^{2\nu-3}$ where $\nu^2=9/4-m_\sigma^2/H^2$ from Eq. (A13) and $\langle\psi_x\bar{\psi}_y\rangle$ scales as r^{-3}, we expect that diagrams that have fewer fermion lines stretched between x and y decrease slower as $r\to\infty$. Thus, we conclude diagram 2(b) gives the dominant contribution to the two-point function.

For diagram 2(b), we expand it using commutators

$$I_b(x,y)=\langle(\bar{\psi}\psi)_{x,r}(\bar{\psi}\psi)_{y,r}\rangle_{c,\mathrm{diag}(b)}\tag{62}$$

$$\begin{aligned}&=4(i\lambda)^2\int^x(dz)\int^y(dw)\langle\bar{\psi}\psi_{[x}\bar{\psi}\psi_{z]}\rangle\langle\bar{\psi}\psi_{[y}\bar{\psi}\psi_{w]}\rangle\langle\sigma_{\{z}\sigma_{w\}}\rangle\\&\quad+4(i\lambda)^2\int^x(dz)\int^y(dw)\langle\bar{\psi}\psi_{\{x}\bar{\psi}\psi_{z\}}\rangle\langle\bar{\psi}\psi_{[y}\bar{\psi}\psi_{w]}\rangle\langle\sigma_{[w}\sigma_{z]}\rangle\Theta(w^0-z^0)\\&\quad+4(i\lambda)^2\int^x(dz)\int^y(dw)\langle\bar{\psi}\psi_{[x}\bar{\psi}\psi_{z]}\rangle\langle\bar{\psi}\psi_{\{y}\bar{\psi}\psi_{w\}}\rangle\langle\sigma_{[z}\sigma_{w]}\rangle\Theta(z^0-w^0)\end{aligned}\tag{63}$$

$$\approx(i\lambda)^2\int^x(dz)\int^y(dw)\langle[\bar{\psi}\psi_x,\bar{\psi}\psi_z]\rangle\langle[\bar{\psi}\psi_y,\bar{\psi}\psi_w]\rangle\langle\sigma_{\{z}\sigma_{w\}}\rangle,\tag{64}$$

where $(dz)=\sqrt{-\det(g_{\mu\nu})}d^4z$, $[\cdots]$ means antisymmetrization and $\{\cdots\}$ means symmetrization, and we have implicitly assumed the PV regulator. From the scalar and fermion mode functions in de Sitter spacetime, we know $\langle[\sigma_{x_1},\sigma_{x_2}]\rangle$ is suppressed by $a^{-2\nu}$ relative to $\langle\{\sigma_{x_1},\sigma_{x_2}\}\rangle$, whereas $\langle[\bar{\psi}\psi_{x_1},\bar{\psi}\psi_{x_2}]\rangle$ is suppressed by a^{-1} relative to $\langle\{\bar{\psi}\psi_{x_1},\bar{\psi}\psi_{x_2}\}\rangle$. The last line is obtained by keeping only the dominant contribution.

Since the fermion particle production ends at t_* and the previously produced particles have been diluted away, we

[15]Since $\langle\delta\rho\delta\rho\rangle$ is frozen as long as the two points are outside of the horizon, we can extrapolate this large spatial separation result obtained at the end of inflation to the recombination time.

expect the z and w integrals to peak around the time t_*. For late time and large spatial separations, the scalar correlator $\langle\sigma_{\{z}\sigma_{w\}}\rangle$ is slowly varying with respect to changes in z and w. Thus we may approximately take $\langle\sigma_{\{z}\sigma_{w\}}\rangle = \langle\sigma_{\{z_0}\sigma_{w_0\}}\rangle$, where $z_0 = (t_*, \vec{x})$ and $w_0 = (t_*, \vec{y})$, and factor it outside of the z, w integral:

$$I_b(x,y) \approx (i\lambda)^2\langle\sigma_{\{z_0}\sigma_{w_0\}}\rangle\left[\int^x (dz)\langle[\bar{\psi}\psi_x, \bar{\psi}\psi_z]\rangle\right] \times\left[\int^y (dw)\langle[\bar{\psi}\psi_y, \bar{\psi}\psi_w]\rangle\right]. \quad (65)$$

The remaining fermion integral $\int^x (dz)\langle[\bar{\psi}\psi_x, \bar{\psi}\psi_z]\rangle$ is quadratically divergent. The counterterms $\delta Z_4\sigma + \delta Z_8 R\sigma$ in $(\bar{\psi}\psi)_r$ are in place to cancel such divergences. Furthermore, our choice of the renormalization conditions given in Sec. III A ensures that a constant shift in σ is equivalent to a shift of the fermion mass [see Eq. (38)]. An explicit computation of the fermion loop integral using the adiabatic subtraction is given in Appendix D. Thus we have

$$\langle(\delta_S)_{r,x}(\delta_S)_{r,y}\rangle_{\rm NLO} \approx \omega_\psi^2\lambda^2[\partial_m \ln n_\psi|_x][\partial_m \ln n_\psi|_y] \times\langle\sigma_{\{(\vec{x},t_*)}\sigma_{(\vec{y},t_*)\}}\rangle, \quad (66)$$

where t_* is the time when fermion production ends [i.e. $m_\psi \sim H(t_*)$] and ∂_m denotes the derivative with respect to m_ψ. Note that $\langle(\delta_S)_{r,x}(\delta_S)_{r,y}\rangle_{\rm NLO}$ freezes for $t > t_*$ since $\partial_m n_\psi$ and n_ψ behave as a^{-3} after the fermion production ends. We will discuss the numerical implications of this result below.

To summarize, we computed the isocurvature correlation function to the next leading order, as in Eq. (66). Intuitively, the light scalar's quantum fluctuation modulates the fermion's mass, which affects the fermion relic abundance. In the same line of thought, we may extrapolate this result to estimate higher order corrections

$$\langle(\delta_S)_{r,x}(\delta_S)_{r,y}\rangle_{\rm full} \approx \omega_\psi^2\frac{\langle n_\psi(m_\psi+\lambda\sigma(\vec{x},t_*))n_\psi(m_\psi+\lambda\sigma(\vec{y},t_*))\rangle_\sigma}{n_\psi^2}, \quad (67)$$

where we have treated n_ψ to be a function of its mass and the expectation value is taken with respect to the σ field.

C. Isocurvature power spectrum

In the long wavelength limit, which corresponds to the low multipoles in the angular CMB anisotropy, the temperature fluctuations dominantly come from the Sach-Wolfe term [18], which is expressed as

$$\frac{\Delta T}{T} = -\frac{1}{5}\zeta - \frac{2}{5}\delta_S. \quad (68)$$

Then the power spectrum of the temperature fluctuations

$$\Delta^2_{\frac{\Delta T}{T}}(k) \equiv \frac{k^3}{2\pi^2}\int d^3x\left\langle\frac{\Delta T}{T}(t,\vec{x})\frac{\Delta T}{T}(t,\vec{0})\right\rangle e^{-i\vec{k}\cdot\vec{x}} = \frac{1}{25}\Delta^2_\zeta(k) + \frac{4}{25}\Delta^2_{\delta_S}(k), \quad (69)$$

$$\Delta^2_\zeta(k) \equiv \frac{k^3}{2\pi^2}\int d^3x\langle\zeta(t,\vec{x})\zeta(t,\vec{0})\rangle e^{-i\vec{k}\cdot\vec{x}}, \quad (70)$$

$$\Delta^2_{\delta_S}(k) \equiv \frac{k^3}{2\pi^2}\int d^3x\langle\delta_S(t,\vec{x})\delta_S(t,\vec{0})\rangle e^{-i\vec{k}\cdot\vec{x}}, \quad (71)$$

where the cross-correlation contribution $\langle\zeta\delta_S\rangle$ has been neglected because of the reason explained in Sec. VII. When the leading term approximation (66) is valid, Eq. (66) yields the isocurvature power spectrum

$$\Delta^2_{\delta_S}(t,k) = \omega_\psi^2(t)\lambda^2\left(\frac{\partial_m n_\psi(m_\psi)}{n_\psi}\right)^2\Delta^2_\sigma(t_*,k) + O(\lambda^4), \quad (72)$$

which includes the extra factor ω_ψ^2 due to the thermal relics. Furthermore, when the mass of scalar field σ is sufficiently light such that σ does not start its coherent oscillation until the fermion particle production ends, i.e., $m_\sigma < H(t_*) < H_{\rm inf}$, the power spectrum for σ is

$$\Delta^2_\sigma(t_*,k) \approx \frac{H^2(t_k)}{4\pi^2}, \quad (73)$$

where t_k is the time when the scale k exits the horizon. Note that we have already shown that the correction of m_σ due to the fermion loop is negligible in Sec. IV. Therefore, the isocurvature power spectrum becomes

$$\Delta^2_{\delta_S}(k) \approx \omega_\psi^2\lambda^2\left(\frac{\partial_m n_\psi(m_\psi)}{n_\psi}\right)^2\frac{H^2(t_k)}{4\pi^2}. \quad (74)$$

The currently known parametric bounds for this isocurvature power spectrum is presented in Sec. VI A.

VI. RESULT AND DISCUSSION

A. Parameter bounds

In this subsection, we present the allowed parameter region in the fermion isocurvature model from the observational constraints using the dark matter relic abundance and the CDM isocurvature power spectrum. In this scenario, there are five independent parameters: m_ψ, $H_{\rm inf}$, λ, $T_{\rm RH}$, and m_σ, where $H_{\rm inf}$ is the Hubble scale during inflation and $T_{\rm RH}$ is the reheating temperature. We assume $H_{\rm inf}$ and $T_{\rm RH}$ are free parameters governed entirely by the inflaton and the reheating sector. As discussed in Sec. II, as long as $m_\sigma \ll m_\psi$, the exact value of the scalar mass m_σ is numerically unimportant in this model. Therefore, we are basically left with two parameters, namely λ and m_ψ.[16]

For the light fermion, $m_\psi < H_{\rm inf}$, the fermion particle number freezes when $H(t_*) \sim m_\psi$ as reviewed in Appendix B. In particular, the Yukawa coupling works effectively as a mass shift in our scenario $m_{\rm eff} = |m_\psi + \lambda\sigma(t_*)|$. The fermion relic abundance (B3) becomes

$$\Omega_\psi h^2 \sim 3r\left(\frac{m_{\rm eff}}{10^{11}\ {\rm GeV}}\right)^2\left(\frac{T_{\rm RH}}{10^9\ {\rm GeV}}\right), \tag{75}$$

where the extra factor r comes from the difference in the effective masses at t_* and later time, at which the energy density of ψ is not negligible, such as the matter dominated (MD) era. For example, if σ is treated as a Gaussian random variable with $\sqrt{\langle\sigma^2\rangle} \sim H_{\rm inf}/2\pi$, we can approximate $r \approx m_\psi/\langle m_{\rm eff}\rangle$ and write

$$\Omega_\psi h^2 \sim \begin{cases} \left(\frac{m_\psi}{10^{11}\ {\rm GeV}}\right)^2\left(\frac{T_{\rm RH}}{10^9\ {\rm GeV}}\right) & \text{if } m_\psi > \lambda H_{\rm inf}/2\pi \\ \frac{2\pi m_\psi}{\lambda H_{\rm inf}}\left(\frac{\lambda H_{\rm inf}}{10^{11}\ {\rm GeV}}\right)^2\left(\frac{T_{\rm RH}}{10^9\ {\rm GeV}}\right) & \text{if } m_\psi < \lambda H_{\rm inf}/2\pi \end{cases}, \tag{76}$$

where $O(1)$ factors are neglected.

Furthermore, from the result (74) in Sec. V C, the fractional isocurvature amplitude [88] becomes

$$\alpha_S \equiv \frac{\Delta^2_{\delta_S}}{\Delta^2_\zeta + \Delta^2_{\delta_S}} \sim \frac{\lambda^2}{2}\left(\frac{m_\psi}{10^4\ {\rm GeV}}\right)^2\left(\frac{H}{10^{13}\ {\rm GeV}}\right)^2\left(\frac{T_{\rm RH}}{10^9\ {\rm GeV}}\right)^2, \tag{77}$$

where we have used

$$\frac{\partial_m n_\psi}{n_\psi} \sim \begin{cases} m_\psi^{-1} & \text{for } m_\psi > \lambda H_{\rm inf}/2\pi \\ 2\pi\lambda^{-1}H_{\rm inf}^{-1} & \text{for } m_\psi < \lambda H_{\rm inf}/2\pi \end{cases}, \tag{78}$$

because the number density n_ψ at the time t_* is determined by only one dimensionful scale $m_{\rm eff} \sim H(t_*)$. The current observational bound [1,89,90] of the isocurvature for the uncorrelated case, i.e. $\langle\zeta\delta_S\rangle = 0$, is $\alpha_S < 0.016$ (95% C.L.) from the Planck + WP9 combined data, which yield the constraints on the parameters λ and m_ψ. Combining the above considerations, we have the parameter plot shown in Fig. 3. We emphasize that the parameter region beyond the

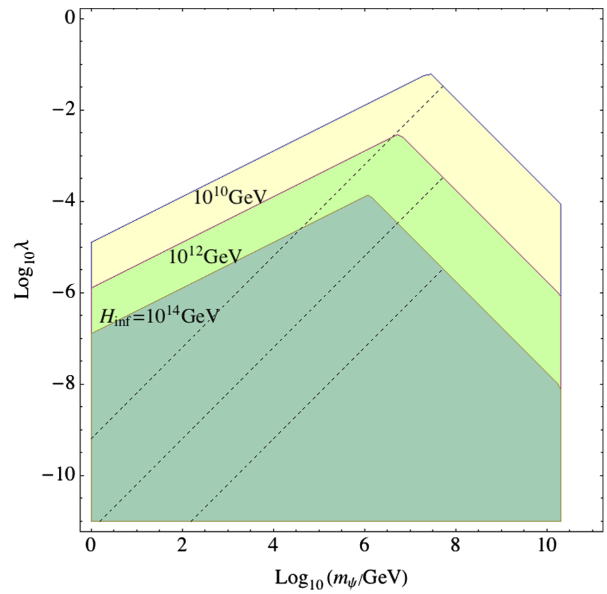

FIG. 3 (color online). Bounds on the fermion mass and Yukawa coupling for various inflationary Hubble scales. The vertical bound corresponds to the total dark matter relic density constraint, and the right diagonal one corresponds to the constraints from the CDM isocurvature, respectively. The left diagonal one is a conservative bound from the σ annihilation of this model, Eq. (50), which may be relaxed. The splitting dashed lines in each region separates the small mass and large mass correction regimes. In this plot, we set $T_{\rm RH} = 10^9$ GeV.

(left diagonal) bound from the σ annihilation, Eq. (50), is not necessarily excluded. Because of the uncertainty of the σ annihilation effect, we provide it as a conservative bound of this model.

The case that $m_\psi < \lambda H_{\rm inf}/(2\pi)$ (which we will refer to as the large mass correction regime) is potentially the most interesting case because the fermion number density n_ψ depends on $|m_\psi + \lambda\sigma|$, not $m_\psi + \lambda\sigma$ as the sign of the fermion mass is irrelevant for particle production.[17] This may lead to interesting features such as large non-Gaussianities when the effective mass varies from negative to positive depending on the local Hubble patches at t_*. However, this parametric region has a couple of problems: (1) the perturbative calculation of n_ψ may be unsuitable since we are not resuming the large mass corrections; and (2) Eq. (46) may not be satisfied. Hence, for the rest of this section, we primarily focus on the case that $m_\psi > \lambda H_{\rm inf}/(2\pi)$, which we will refer to as the small mass correction regime.

B. Non-Gaussianities

In this subsection, we compute the bispectrum $B_S(\vec{p}_1, \vec{p}_2, \vec{p}_3)$ defined by

[16]Note that we implicitly assume that if m_ψ and $T_{\rm RH}$ are such that the dark matter relic abundance is not saturated by the ψ energy density, the other CDM sector in Eq. (6) is adjusted to provide the rest of the dark matter. Note that when the ψ dark matter abundance is small, no large tuning is needed to make this occur since the well known WIMP miracle can saturate the dark matter abundance.

[17]The sign of the fermion mass changes under a chiral transformation.

$$(2\pi)^3\delta^{(3)}\left(\sum_i \vec{p}_i\right)B_S(\vec{p}_1,\vec{p}_2,\vec{p}_3) = \int d^3x_1 d^3x_2 d^3x_3 e^{-i\sum_i \vec{p}_i\cdot\vec{x}_i}\langle\delta_S(\vec{x}_1)\delta_S(\vec{x}_2)\delta_S(\vec{x}_3)\rangle. \tag{79}$$

The fermion density fluctuation is intrinsically non-Gaussian since n_ψ is the nonlinear function of σ, which is treated as a Gaussian random variable. When the effective mass fluctuation due to $\lambda\sigma$ is small, we can Taylor expand the number density with respect to $\lambda\sigma$,

$$n_\psi(m_\psi+\lambda\sigma) = n_\psi(m_\psi)+\lambda(\partial_{m_\psi}n_\psi(m_\psi))\sigma + \frac{1}{2}\lambda^2(\partial^2_{m_\psi}n_\psi(m_\psi))\sigma^2+O(\lambda^3). \tag{80}$$

Then the bispectrum is written as

$$B_S(\vec{p}_1,\vec{p}_2,\vec{p}_3) = \lambda^4\omega_\psi^3\frac{(\partial_m n_\psi)^2(\partial_m^2 n_\psi)}{n_\psi^3}[\Delta_\sigma^2(p_1)\Delta_\sigma^2(p_2)+2\,\text{perms}] + O(\lambda^6), \tag{81}$$

which is shown diagrammatically in Fig. 4. Now we compare this with the observational non-Gaussianities using the conventional non-Gaussian parameter $f_{\rm NL}$ defined by

$$B_\zeta(\vec{p}_1,\vec{p}_2,\vec{p}_3) \equiv \frac{6}{5}f_{\rm NL}[\Delta_\zeta^2(p_1)\Delta_\zeta^2(p_2)+2\,\text{perms}]. \tag{82}$$

Identifying B_ζ as the bispectrum of the temperature fluctuation using Eq. (68) and comparing it with B_S, we find in the squeezed triangle limit

$$f^S_{\rm NL} = \frac{8B_S}{B_\zeta|_{f_{\rm NL}=1}} = 8\frac{5}{6}\lambda^4\omega_\psi^3\frac{(\partial_m n_\psi)^2(\partial_m^2 n_\psi)}{n_\psi^3}\frac{\Delta_\sigma^2(p_1)\Delta_\sigma^2(p_2)+2\text{ perms}}{\Delta_\zeta^2(p_1)\Delta_\zeta^2(p_2)+2\text{ perms}}. \tag{83}$$

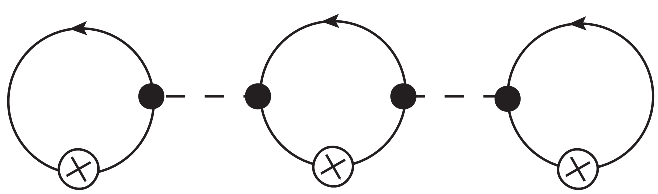

FIG. 4. The leading order diagrams to 3-point function $\langle\delta_S\delta_S\delta_S\rangle$ is shown. The cross-dotted vertices corresponds to $\bar{\psi}\psi/n_\psi$ insertions.

The factor of 8 arises because the radiation transfer function for isocurvature is twice larger than that for adiabatic perturbation for the low multipoles of the CMB anisotropy as shown in Eq. (68). Although the isocurvature non-Gaussianities parameter $f^S_{\rm NL}$ should not be compared directly with $f_{\rm NL}$ defined by the curvature perturbation [91], this can be done with the extra $O(1)$ correction factor [31,34,74,92–94]. The reason why $\partial_m^2 n_\psi$ appears instead of a first derivative is because the squeezed triangle limit allows the short distance propagator to become important. Furthermore, subhorizon physics via the Yukawa interaction, in principle, gives rise to the non-Gaussianities of other types, e.g., the equilateral type. We postpone this study for future work.

In order to obtain the functional structure of $n_\psi(m,H;t)$, which relies on the background behavior, we specialize to the case of the inflaton coherent oscillation reheating scenarios, in which the total fermion number freezes during the reheating. During the early stage of the reheating when the inflaton field oscillates coherently, the equation of state of the inflaton is zero and the background behaves like the MD era. After approximating the early stage of the reheating to the MD-like era (i.e. inflaton coherent oscillations period), we get [see Eq. (B2)]

$$n_\psi(t)\sim\frac{m_\psi^3}{3\pi^2}\left(\frac{a(t_m)}{a_t}\right)^3\sim m_\psi H_e^2\left(\frac{a_e}{a_t}\right)^3. \tag{84}$$

However, this leading order result gives $\partial^2_{m_\psi}n_\psi=0$ which renders $f^S_{\rm NL}=0$ via Eq. (83).

To find the nonzero result of $f^S_{\rm NL}$, we need to study the mass dependence of n_ψ in more detail, which in turn requires the knowledge of $|\beta_k(t;m)|^2$. To this point, we have approximated our spectrum by $|\beta_k(t;m)|^2\sim 1/2\Theta(k_*-k)$, where $k_*=a(t_*)m$ and t_* is the time when $m=H$. However, in general the spectrum should contain more than one characteristic scale, such as $k_e=a(t_e)H_e$ where t_e marks the end of inflation. Thus, in general, the number density should contain a correction factor $f(\frac{m}{H_e})$, i.e.

$$n_\psi\sim m_\psi H_e^2\left(\frac{a_e}{a_t}\right)^3 f\left(\frac{m_\psi}{H_e}\right) \tag{85}$$

and $f(0)=1$. This higher order correction to n_ψ would render $\partial_m^2 n_\psi\neq 0$ for the MD-like reheating scenario.

For simplicity, if we assume that $f(x)=1+a_1x$,[18] then in the limit where Δ_σ^2, Δ_ζ^2, and $\Delta_{\delta_S}^2$ are scale invariant, we find

[18]On very general grounds, n_ψ cuts off exponentially at very large masses, $m_\psi\gtrsim H_e$, as shown in Appendix B and Refs. [30,31,51,52,78]. From this, we qualitatively estimate the correction factor f from this exponential cutoff, which gives an $O(1)$ value for a_1.

$$f_{\rm NL}^S \sim a_1\left(\frac{\alpha_S(\lambda, m_\psi, H_e, T_{\rm RH})}{0.02}\right)^2\left(\frac{\Omega_\psi h^2(m_\psi, T_{\rm RH})}{10^{-7}}\right)^{-1} \times\left(\frac{m_\psi/H_e}{10^{-1}}\right). \quad (86)$$

Although we would naively guess $a_1 \sim O(1)$, the justification of the Taylor expansion for $f(x)$ and the estimation of the coefficient a_1 will be left for future work since the main thrust of this work is the computation of isocurvature perturbations and not the non-Gaussianities. The maximum $f_{\rm NL}$ for the $m_\psi \gtrsim \lambda H_{\rm inf}/(2\pi)$ case (consistent with the small mass correction case) is achieved when this inequality is saturated and α_S is at its phenomenological maximum. We find this maximum to be at

$$f_{NL,\max}^S \sim O(100)a_1\frac{m_\psi}{H_{\rm inf}/(2\pi)}. \quad (87)$$

Recall that our scenario assumes that $2\pi m_\psi/H_{\rm inf} < 1$. Hence, although $f_{\rm NL}^S$ cannot be made arbitrarily large, there may exist a parametric regime in which $f_{\rm NL}^S$ is observable depending on a_1. Note that this extremum value corresponds to making the inhomogeneities $O(1)$ while staying consistent with phenomenology through the ω_ψ dilution factor: i.e. at this parametric point, the fermion abundance is $\Omega_\psi h^2 \approx 10^{-6}$ while most of the CDM is made up of assumed dark matter different from ψ.

VII. NATURAL SUPPRESSION OF GRAVITATIONAL COUPLING TO THE INFLATON

As briefly discussed in Sec. II, the gravity induced coupling of the fermion to the inflaton gives a suppressed contribution to the isocurvature correlation function. We would like to consider this in more detail in this section. In addition, the argument below also shows that $\langle\bar\psi\psi\zeta\rangle$ cross-correlation is negligible, justifying the classification of these fermionic isocurvature perturbations as uncorrelated.

First, consider the $\zeta\psi\psi$ interaction given by Eq. (F17) following the argument given in Ref. [53]. In this case, the most important coupling term is $a^2\zeta\delta_{ij}T_\psi^{ij} \in \mathcal{H}_{\rm int}$ because the other interactions are derivatively suppressed, and it decays as $O(1/a^2)$ or faster. Since ζ also freezes outside the horizon, using the similar argument given surrounding Eq. (65) we can factor the ζ correlation function out of the dominantly contributing integral, which corresponds to the diagram (b) in Fig. 2. Then we have

$$\begin{aligned} I_{\zeta\psi\psi}(x,y) &\approx (i)^2\langle\zeta_{\{z_0}\zeta_{w_0\}}\rangle\left[\int_{t_r}^t dt_z\int d^3za^3(t_z)\langle[\bar\psi\psi_x, T_{\psi i}^i(z)]\rangle\right] \\ &\quad\times\left[\int_{t_r}^t dt_w\int d^3wa^3(t_w)\langle[\bar\psi\psi_y, T_{\psi i}^i(w)]\rangle\right] \\ &\quad + O\left(\frac{a^2(t_r)}{a^2(t)}\right), \end{aligned} \quad (88)$$

where $z_0 = (t_*, \vec x)$, $w_0 = (t_*, \vec y)$, $t = x^0 = y^0$, and t_r denotes the time that the comoving distance $r = |\vec x - \vec y|$ crosses the horizon during inflation. In the integral, we have assumed the PV regulator. Note that $\lambda\int(dz)T_{\psi i}^i$ is a generator of the spatial dilatation, $x^i \to (1+\lambda)x^i$ which is an element of diffeomorphism. Thus, we have

$$\int_{-\infty}^t dt_z\int d^3za^3(t_z)\langle[\bar\psi\psi_x, T_{\psi i}^i(z)]\rangle = 0 \quad (89)$$

because $\bar\psi\psi$ is a diffeomorphism invariant scalar. Indeed, this is a Ward identity similar to that of Ref. [53]. Although the integral in Eq. (88) does not completely vanish (because of the time integral limit being t_r and not $-\infty$), the mode function of ψ decays as $1/a^3$ (as shown in Appendix C) because of the classical conformal symmetry characterizing the massless fermionic sector,[19] and we have

$$\int_{t_r}^t dt_z\int d^3za^3(t_z)\langle[\bar\psi\psi_x, T_{\psi i}^i(z)]\rangle \sim O\left(\frac{a^3(t_r)}{a^3(t)}\right). \quad (90)$$

In a similar manner, we can have

$$\langle\zeta_x(\bar\psi\psi)_y\rangle \sim O\left(\frac{a^2(t_r)}{a^2(t)}\right). \quad (91)$$

Therefore, we can conclude that large scale density perturbations of ψ particles generated by ζ interaction and the curvature and isocurvature cross-correlation via the $\zeta\bar\psi\psi$ are negligible.

VIII. SUMMARY AND CONCLUSION

In this work, we have presented a fermionic isocurvature scenario which contains fermionic field fluctuation information during inflation. To our knowledge, this is the first work that describes isocurvature inhomogeneities of

[19] Thus, the result is different for a scalar case, which is minimally coupled to gravity. In particular, the cross-correlation for the light scalar case is computed in Ref. [53] and is

$$\langle\zeta(t,\vec x)\sigma^2(t,\vec y)\rangle \sim O\left(\left(\frac{a(t_r)}{a(t)}\right)^{3-2\nu}, \left(\frac{a(t_r)}{a(t)}\right)^2\right),$$

where $\nu \equiv \sqrt{\frac{9}{4} - \frac{m_\sigma^2}{H^2}}$.

fermionic fields during inflation. Because massless free fermions have a tree-level conformal symmetry, such isocurvature models must couple to a conformal symmetry breaking sector. Because the ζ sector coupling to fermion ψ is suppressed due to the dilatation symmetry, an additional scalar sector σ is coupled to ψ (with mass m_ψ) through a Yukawa coupling with strength λ. Composite operator renormalization in curved spacetime plays an important role in determining the isocurvature perturbations. We have computed the fermion isocurvature two point correlation function which has its dominant contribution in the long wavelength limit coming at one-loop 1PI level. We have also estimated the local non-Gaussianity and found a value that is promising for observability for a particular corner of the parameter space.

As far as the existence proof inspired "minimal" model of this paper is concerned, a large phenomenologically viable parameter region spanned by $\{\lambda, m_\psi\}$ exists for various inflationary models controlled by $\{H_{\rm inf}, T_{\rm RH}\}$. The large λ parameter region is bounded either by current CMB constraints on isocurvature perturbations or by the constraint of σ not decaying to ψ. The large m_ψ region is constrained by the relic abundance nonoverclosure. The small m_ψ region is constrained by requiring that σ not decay to ψ (for a fixed λ and $H_{\rm inf}$). The large non-Gaussianity parametric region is associated with the largest λ consistent with isocurvature bounds and the simplifying assumption $m_\psi \gtrsim \lambda H_{\rm inf}/(2\pi)$. This intuitively corresponds to a large fermion inhomogeneity [i.e. $\delta\rho_\psi/\bar{\rho}_\psi \sim O(1)$] with a tiny $\bar{\rho}_\psi/(\bar{\rho}_\psi + \bar{\rho}_m)$ where $\bar{\rho}_m$ corresponds to an adiabatic cold dark matter component that helps saturate the phenomenologically measured cold dark matter abundance.

Our results regarding the gravitational fermion production give good dynamical intuition on many models with dynamical fermions existing during inflation. One shortcoming of the explicit model used in the current work is the tuning of the σ sector imposed to keep it light and to prevent any σ decay into ψ. In addition to model building issues, it would be interesting to consider in the future non-Gaussianities from such models more completely and carefully beyond the estimation presented in this work. It may also be interesting to see what UV model fermionic sector built independently of cosmological motivation can be constrained using the analysis presented in this paper.

ACKNOWLEDGMENTS

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APPENDIX A: SCALAR AND SPINOR FIELDS IN CURVED SPACETIME

First we list the relevant results about scalar field. Consider the following action:

$$S = \int d^4x\sqrt{|g|}\left\{-\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\xi R\phi^2\right\}. \tag{A1}$$

This gives rises to equation of motion

$$\frac{1}{\sqrt{|g|}}\partial_\mu(g^{\mu\nu}\sqrt{|g|}\partial_\nu\phi) - (m^2 + \xi R)\phi = 0. \tag{A2}$$

The scalar product between two solutions is defined as

$$(\phi_1, \phi_2) = -i\int_\Sigma [\phi_1\partial_\mu\phi_2^* - \phi_2\partial_\mu\phi_1^*]\sqrt{|g_\Sigma|}d\Sigma^\mu, \tag{A3}$$

where Σ is a spacelike hypersurface.

For the FRW metric, we can use mode decomposition

$$\phi(x) = \int d^3k(c_{\vec{k}}u_{\vec{k}}(x) + c_{\vec{k}}^\dagger u_{\vec{k}}^*(x)) \tag{A4}$$

with the normalization condition

$$[c_{\vec{k}}, c_{\vec{p}}^\dagger] = \delta^3(\vec{k} - \vec{p}), \tag{A5}$$

$$(u_{\vec{k}}, u_{\vec{p}}) = \delta^3(\vec{k} - \vec{p}). \tag{A6}$$

The mode functions can be written explicitly as

$$u_{\vec{k}}(x) = \frac{e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^{3/2}a(\eta)}f_k(\eta), \tag{A7}$$

$$f_k\partial_\eta f_k^* - f_k^*\partial_\eta f_k = i. \tag{A8}$$

The time part of the mode function obeys the differential equation

$$\frac{d^2}{d\eta^2}f_{k,\eta} + \left\{k^2 + a_\eta^2\left[m^2 + \left(\xi - \frac{1}{6}R(\eta)\right)\right]\right\}f_{k,\eta} = 0, \tag{A9}$$

where $R(\eta) = 6a^{-1}\partial_\eta^2 a$, and η is the conformal time. For de Sitter spacetime, the mode solution for a minimally coupled scalar $(\xi = 0)$ is

$$f_k(\eta) = \frac{1}{\sqrt{2k}}\sqrt{\frac{\pi}{2}\left(\frac{k}{aH}\right)}e^{i\frac{\pi}{2}(\nu+\frac{1}{2})}H_\nu^{(1)}\left(\frac{k}{aH}\right), \tag{A10}$$

where $\nu^2 = \frac{9}{4} - \frac{m^2}{H^2}$.

The following relations of the first kind of Hankel functions are useful:

$$H_\nu^{(1)}(z) \to -i\frac{\Gamma(\nu)}{\pi}\left(\frac{2}{z}\right)^\nu \quad (z\to 0), \tag{A11}$$

$$H_\nu^{(1)}(z) \to \sqrt{\frac{2}{\pi z}}e^{-i\frac{\pi}{2}(\nu+\frac{1}{2})}e^{iz} \quad (z\to\infty). \tag{A12}$$

From the mode expansion, we may construct the equal-time correlator in dS spacetime. In particular, we are interested in the large separation limit. For light scalar, when ν is real, we have

$$\langle\sigma_x\sigma_y\rangle \approx \frac{H^2}{8\pi}\frac{\Gamma(\frac{3}{2}-\nu)}{\Gamma(\frac{3}{2})\Gamma(1-\nu)\sin(\nu\pi)}(aHr)^{2\nu-3}. \tag{A13}$$

For heavy scalar, when $\nu = i\alpha$ and if $\alpha \sim \frac{m}{H} \gg 1$, then

$$\langle\sigma_x\sigma_y\rangle \approx \frac{H^{3/2}m^{1/2}}{\pi^{3/2}}e^{-\frac{m}{H}\pi}\sin\left[2\frac{m}{H}\ln(aHr)-\frac{1}{4}\pi\right](aHr)^{-3}. \tag{A14}$$

Next, we give the result for the spinor field. Consider the free Dirac field ψ action

$$S = \int(dx)(i\bar{\psi}\gamma^\mu\nabla_\mu\psi - m\bar{\psi}\psi), \tag{A15}$$

where $(dx) = d^4x\sqrt{|g_x|}$ and $\gamma^\mu \equiv \gamma^a e_a^\mu$ with vierbein e_a^μ. The covariant derivatives for ψ are defined by

$$\nabla_\mu\psi = \partial_\mu\psi + \frac{1}{2}\omega_\mu^{ab}\Sigma_{ab}\psi, \tag{A16}$$

the spin-connection is defined by

$$\omega_\mu^{ab} = e_\nu^a\nabla_\mu e^{b\nu}, \tag{A17}$$

and the Lorentz generator on the spinor field is given by

$$\Sigma_{ab} = -\frac{1}{4}[\gamma_a,\gamma_b], \tag{A18}$$

where the γ matrices satisfy $\{\gamma_a,\gamma_b\} = -2\eta_{ab}$ with $\eta \equiv \mathrm{diag}(-1,1,1,1)$. Note that the sign convention is chosen such that $[\Sigma^{12},\Sigma^{23}] = \Sigma^{13}$.

Extremizing the action with respect to $\delta\bar{\psi}$ and $\delta\psi$ yields the equations of motion:

$$(i\gamma^\mu\nabla_\mu - m)\psi = 0, \qquad \nabla_\mu\bar{\psi}(-i\gamma^\mu) - \bar{\psi}m = 0. \tag{A19}$$

The solution space can be endowed with a scalar product as

$$(\psi_1,\psi_2)_\Sigma = \int d\Sigma n_\mu\bar{\psi}_1\gamma^\mu\psi_2 \tag{A20}$$

in which Σ is an arbitrary spacelike hypersurface, $d\Sigma$ is the volume 3-form on this hypersurface computed with the induced metric, and n_μ is the future-pointing timelike unit vector normal to Σ. The current conservation condition

$$\nabla_\mu(\bar{\psi}_1\gamma^\mu\psi_2) = 0 \tag{A21}$$

implies the integral in the scalar product is independent of the choice of Σ.

If we adopt the Dirac basis for the γ matrices, i.e.

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I\end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0\end{pmatrix}, \tag{A22}$$

the mode functions can be written as

$$U_{\vec{k},r}(x) = \frac{1}{a_x^{3/2}}\frac{e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^{3/2}}\begin{pmatrix} u_{A,k,x^0} \\ ru_{B,k,x^0}\end{pmatrix}\otimes h_{\hat{k},r}, \tag{A23}$$

$$\begin{aligned} V_{\vec{k},r}(x) &= -i\gamma^2 U^*_{\vec{k},r}(x) \\ &= \frac{1}{a_x^{3/2}}\frac{e^{-i\vec{k}\cdot\vec{x}}}{(2\pi)^{3/2}}\begin{pmatrix} ru^*_{B,k,x^0} \\ -u^*_{A,k,x^0}\end{pmatrix}\otimes(-i\sigma_2)h^*_{\hat{k},r}, \end{aligned} \tag{A24}$$

where $h_{\hat{k},r}$ is the eigenvector of $\hat{k}\cdot\vec{\sigma}$. The normalization conditions require

$$h^\dagger_{\hat{k},r}h_{\hat{k},s} = \delta_{rs}, \tag{A25}$$

$$|u_{A,k,\eta}|^2 + |u_{B,k,\eta}|^2 = 1. \tag{A26}$$

The time dependent parts of the mode functions obey the following equation:

$$i\frac{d}{d\eta}\begin{pmatrix} u_A \\ u_B\end{pmatrix} = \begin{pmatrix} am & k \\ k & -am\end{pmatrix}\begin{pmatrix} u_A \\ u_B\end{pmatrix}. \tag{A27}$$

In the special case of the de Sitter background with the Bunch-Davies boundary condition, we have

$$\begin{pmatrix} u_A \\ u_B\end{pmatrix}^{in}_{k,\eta} = \begin{pmatrix} \sqrt{\frac{\pi}{4}(\frac{k}{aH_e})}e^{i\frac{\pi}{2}(1-i\frac{m}{H_e})}H^{(1)}_{\frac{1}{2}-i\frac{m}{H_e}}(\frac{k}{aH}) \\ \sqrt{\frac{\pi}{4}(\frac{k}{aH_e})}e^{i\frac{\pi}{2}(1+i\frac{m}{H})}H^{(1)}_{\frac{1}{2}+i\frac{m}{H_e}}(\frac{k}{aH})\end{pmatrix} \tag{A28}$$

$$\text{if}|kx^0| \ll 1 \longrightarrow \begin{pmatrix} \frac{1}{\sqrt{2\pi}}e^{\frac{\pi m}{2H}}e^{-im(t-t_e)+i\frac{m}{H}\ln(2k/a_eH)}\Gamma(\frac{1}{2}-i\frac{m}{H}) \\ \frac{1}{\sqrt{2\pi}}e^{-\frac{\pi m}{2H}}e^{+im(t-t_e)-i\frac{m}{H}\ln(2k/a_eH)}\Gamma(\frac{1}{2}+i\frac{m}{H})\end{pmatrix}. \tag{A29}$$

Since the interaction picture operator $\psi(x)$ obeys the same classical equations, Eq. (A19), we can expand the operator using $\{U_i, V_i\}$ as the basis,

$$\psi(x) = \sum_i a_i U_i(x) + b_i^\dagger V_i(x), \tag{A30}$$

and the normalization conditions on U_i, V_i give the usual canonical anticommutation relations of the creation and annihilation operators.

The first order WKB approximation is defined as

$$\begin{pmatrix} u_A \\ u_B \end{pmatrix}^{\mathrm{WKB}}_{k,\eta} = \begin{pmatrix} \sqrt{\frac{\omega+am}{2\omega}} \\ \sqrt{\frac{\omega-am}{2\omega}} \end{pmatrix} e^{-i\int^\eta \omega d\eta'}. \tag{A31}$$

In the following, when we talk about the fermion particle, we are implicitly referring to the WKB mode.

Thus one can introduce the time-dependent Bogoliubov coefficients $\{\alpha_{k,\eta}, \beta_{k,\eta}\}$ between the in modes and WKB modes:

$$\begin{pmatrix} u_A \\ u_B \end{pmatrix}^{\mathrm{in}}_{k,\eta} = \alpha_{k,\eta} \begin{pmatrix} u_A \\ u_B \end{pmatrix}^{\mathrm{WKB}}_{k,\eta} + \beta_{k,\eta} \begin{pmatrix} u_B^* \\ -u_A^* \end{pmatrix}^{\mathrm{WKB}}. \tag{A32}$$

Clearly, $(\alpha, \beta) \to (1, 0)$ as $\eta \to -\infty$. We may also note that the Bogoliubov coefficients obey the normalization condition as

$$|\alpha_{k,\eta}|^2 + |\beta_{k,\eta}|^2 = 1 \tag{A33}$$

in agreement with fermion statistics.

Using Eqs. (A32), (A31), and (A27), we can derive the evolution equation for the Bogoliubov coefficients, as shown in Eq. (B1).

APPENDIX B: REVIEW OF FERMION PARTICLE PRODUCTION

In this section, we give a brief review of the main result about fermion production during inflation [51]. The fermion number density can be obtained by solving these equations of Bogoliubov coefficients

$$\partial_\eta \begin{pmatrix} \alpha_{k,\eta} \\ \beta_{k,\eta} \end{pmatrix} = \frac{a^2 mkH}{2\omega^2} \begin{pmatrix} 0 & e^{2i\int^\eta \omega d\eta'} \\ -e^{-i\int^\eta \omega d\eta'} & 0 \end{pmatrix} \begin{pmatrix} \alpha_{k,\eta} \\ \beta_{k,\eta} \end{pmatrix}. \tag{B1}$$

We define the nonadiabaticity for a mode k as $\epsilon_{k,\eta} = \frac{mk_pH}{\omega_p^3}$, where subscript p stand for "physical," $\omega_p = \omega/a$, etc. As the system evolves from an initial vacuum condition of $\binom{\alpha_{k,\eta}}{\beta_{k,\eta}} = \binom{1}{0}$, $\beta_{k,\eta}$ will only increase significantly when $\epsilon_{k,\eta} \sim O(1)$. This implies the following results:

(1) In the heavy mass limit ($m_\psi \gg H_{\mathrm{inf}}$), $\epsilon_{k,\eta}$ is always suppressed by $\frac{H}{m_\psi}$, and we get $|\beta_{k,\eta}|^2 \sim \exp[-C\frac{m_\psi}{H(\eta_k)}] \ll 1$, where C is some order one constant and $H(\eta_k)$ is the Hubble rate at the most nonadiabatic moment for mode k.

(2) In the light mass limit ($m_\psi \ll H_{\mathrm{inf}}$), $\epsilon_{k,\eta}$ is largest when $k_p \sim m_\psi$, and we call this time η_k. If $m_\psi < H(\eta_k)$, we have $|\beta_k|^2 \sim \frac{1}{2}$; otherwise it is suppressed by $\exp[-C\frac{m_\psi}{H(\eta_k)}]$ as well.

Since the heavy fermion production is always exponentially suppressed by the m_ψ/H ratio, we focus on the light fermion case. The energy density at time t is given by

$$\rho(t) \sim \frac{m_\psi^4}{3\pi^2} \left(\frac{a(t_*)}{a(t)}\right)^3, \tag{B2}$$

where t_* is the time when $H(t) = m_\psi$. If t_* occurs during reheating, one gets the relic abundance today time as

$$\Omega_\psi h^2 \sim 3 \left(\frac{m_\psi}{10^{11}\ \mathrm{GeV}}\right)^2 \left(\frac{T_{\mathrm{RH}}}{10^9\ \mathrm{GeV}}\right). \tag{B3}$$

APPENDIX C: ASYMPTOTIC BEHAVIOR OF $\langle\psi_x\bar{\psi}_y\rangle$ AT LARGE r

In this section we derive the result about the leading order contribution to $\langle n_{\psi,x} n_{\psi,y}\rangle$, i.e. Eq. (52). By Wick contraction, this reduces to computing the field correlator $\langle\psi_x\bar{\psi}_y\rangle$. The standard way to compute the correlator is to plug in the mode decomposition Eq. (A30) and compute the mode functions $\{U_i, V_i\}$. The difficulties lie in how to obtain the mode functions on a curved spacetime. For inflationary background spacetime, one can use the de Sitter spacetime as an approximation and obtain exact analytic solutions. However, it is unclear how these mode solutions evolve after inflation ends. Such postinflationary solutions are relevant for our computation because the particle production freezes out after the end of inflation. Here we give an approach that answers this question.

First, we plug in the mode decomposition to the equal-time correlator:

$$\langle\psi_x\bar{\psi}_y\rangle = \int d^3k \frac{1}{a_x^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^3} \begin{pmatrix} |u_{A,k,x^0}|^2 \otimes I_2 & -u_{A,k,x^0} u^*_{B,k,x^0} \otimes (\hat{k}\cdot\vec{\sigma}) \\ u_{B,k,x^0} u^*_{A,k,x^0} \otimes (\hat{k}\cdot\vec{\sigma}) & -|u_{B,k,x^0}|^2 \otimes I_2 \end{pmatrix}, \tag{C1}$$

where we have performed the spin sum in the last step. Since

$$\int d^3k \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^3}|u_{A,k,x^0}|^2 = \int d^3k \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^3}(1-|u_{B,k,x^0}|^2) \quad \text{(C2)}$$

$$= \delta^3(\vec{r}) - \int d^3k \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^3}|u_{B,k,x^0}|^2 \quad \text{(C3)}$$

and $\vec{r} \neq 0$, we see the diagonal elements are the same. Then we perform the angular integral $d^2\hat{k}$. Recall that

$$\int d^3k e^{i\vec{k}\cdot\vec{r}} f(k) = \int 4\pi k^2 dk \frac{\sin(kr)}{kr} f(k), \quad \text{(C4)}$$

$$\int d^3k e^{i\vec{k}\cdot\vec{r}} \hat{k}_i f(k) = (-i\hat{r}_i \partial_r) \int 4\pi k^2 dk \frac{\sin(kr)}{kr}\frac{f(k)}{k}. \quad \text{(C5)}$$

After the angular integral, we have

$$\langle \psi_x \bar{\psi}_y \rangle = \int \frac{4\pi k^2 dk}{(2\pi)^3}\begin{pmatrix} A & B \\ B^* & C \end{pmatrix}, \quad \text{(C6)}$$

$$A = |u_{A,k,\eta}|^2 \cdot \frac{\sin(kr)}{kr}, \quad \text{(C7)}$$

$$B = (i\hat{r}\cdot\vec{\sigma})u_{A,k,\eta}u^*_{B,k,\eta} \cdot \partial_r \frac{\sin(kr)}{kr}\frac{1}{k}, \quad \text{(C8)}$$

$$C = -|u_{B,k,\eta}|^2 \cdot \frac{\sin(kr)}{kr}. \quad \text{(C9)}$$

It is sufficient to study these two integrals for the diagonal and off-diagonal elements,

$$I_{11} = I_{22} = \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3}|u_{A,k,\eta}|^2 \cdot \frac{\sin(kr)}{kr}, \quad \text{(C10)}$$

$$I_{12} = I^*_{21} = \partial_r \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3} u_{A,k,\eta}u^*_{B,k,\eta}\frac{\sin(kr)}{kr}\frac{1}{k}. \quad \text{(C11)}$$

Now, we only need to find the mode function u_A, u_B, and perform the mode sum.

Let us consider the mode functions first. Since we are interested in evaluating the fermion field correlator at a time when the fermion production has ended, i.e. when $m \gg H(x^0)$ and is in the limit $r \to \infty$, we can make the following approximations about the mode functions $\{u_{A,k,x^0}, u_{B,k,x^0}\}$. First, since the particle production has stopped, the nonadiabatic parameter is suppressed by $\frac{H(t)}{m}$, and thus we can approximately replace the Bogoliubov coefficients by their late time asymptotic values, i.e.

$$\alpha_{k,x^0} \approx \alpha_k, \qquad \beta_{k,x^0} \approx \beta_k. \quad \text{(C12)}$$

Second, since we want to capture the particle production effect on the correlator and the produced particles are nonrelativistic at the time of production, by the time x^0 which is sufficiently long after the production has ended, we may approximate the produced modes that all have $k \ll a(x^0)m$. Thus, the WKB modes can be approximated by

$$\begin{pmatrix} u_A \\ u_B \end{pmatrix}^{\text{WKB}}_{k,\eta,IR} = \begin{pmatrix} \sqrt{\frac{\omega+am}{2\omega}} \\ \sqrt{\frac{\omega-am}{2\omega}} \end{pmatrix} e^{-i\int^\eta \omega d\eta'} \to \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} e^{-i\int^\eta \omega d\eta'}. \quad \text{(C13)}$$

Combining these two approximations, we have

$$\begin{pmatrix} u_A \\ u_B \end{pmatrix}^{in}_{k,\eta,IR} \approx \begin{pmatrix} \alpha_k \frac{1}{\sqrt{2}} e^{-i\int^\eta \omega d\eta'} \\ -\beta_k \frac{1}{\sqrt{2}} e^{i\int^\eta \omega d\eta'} \end{pmatrix}. \quad \text{(C14)}$$

Thus we can easily evaluate I_{11}, I_{12}:

$$2\pi^2 I_{11,IR} = \frac{1}{r}\text{Im}\int_0^\infty kdk\frac{1}{2}[1-n(k)]\cdot e^{ikr}. \quad \text{(C15)}$$

We note that for the contribution from 1 vanishes

$$\frac{1}{r}\text{Im}\int_0^\infty kdk[1]\cdot e^{ikr} = \frac{1}{r}\text{Im}\int_0^\infty (is)ids[1]\cdot e^{-sr} = 0. \quad \text{(C16)}$$

For the contribution from $n(k)$, we may assume it to be a real analytic function on $\mathbb{R}^+$ and can be analytically continuated to the upper-right quadrant of the complex k plane. The location of the singularity of $n(k)$ determines the contour of k. For example, we may consider the $n(k)$ for the heavy fermion case ($m > H_{\text{inf}}$):

$$n(k)_{\text{heavy}} = \exp\left[-\frac{4(k/a_{\text{nad}})^2}{mH} - \frac{4m}{H}\right], \quad \text{(C17)}$$

where a_{nad} is at the nonadiabatic time point. In this case, the nonadiabatic time is the transition from the de Sitter era to the reheating era, i.e. $a_{\text{nad}} = a_e$. One can apply the steepest descent to find that

$$2\pi^2 I_{11,\text{heavy},IR} \approx -\frac{1}{r}\exp\left[-\frac{4m}{H} - \frac{1}{16}mHr^2\right](a_e^2 mH) \times \text{Im}\left[-i\frac{1}{4}\sqrt{mH}a_e r\frac{1}{2}\sqrt{\pi}\right] \quad \text{(C18)}$$

$$= \frac{1}{8}\sqrt{\pi}a_e^3(mH)^{\frac{3}{2}}\exp\left[-\frac{4m}{H}-\frac{1}{16}a_e^2 mHr^2\right]. \quad (C19)$$

For the light fermion, we may approximate the number density spectrum as

$$n(k)_{\text{light}} = \frac{1}{1+\exp(\frac{k^2}{(a_{\text{nad}}m)^2})}, \quad (C20)$$

where the nonadiabatic point occurs when H drops below m, i.e. $a_{\text{nad}} = a(\eta_*) = a_*$. This ansatz is only used to mimic the cutoff of the spectrum at $k \sim a_{\text{nad}}m$. The singularity lies at

$$\frac{k^2}{a_*^2 m^2} = (2n+1)\pi i, \quad n = 0, 1, 2, \ldots, \quad (C21)$$

or $k_{*,n} = a_* m\sqrt{(2n+1)\pi}e^{\frac{\pi}{4}i}$. Again, one can perform the steepest descent around the $n = 0$ singularity $k_* = a_* m\sqrt{\pi}e^{\frac{\pi}{4}i}$. Let $\delta = (k-k_*)/a_* m$, and we have

$$2\pi^2 I_{11,\text{light},IR} = \pi a_*^3 \frac{m^2}{a_* r}\exp\left[-\sqrt{\frac{\pi}{2}}a_* mr\right]\cos\left(\sqrt{\frac{\pi}{2}}a_* mr\right). \quad (C22)$$

For both the heavy and the light fermion cases, $I_{11} \propto \exp(-a_* Mr)$, where $a_* M$ is the scale that $n(k)$ cuts off. We should also remind ourselves that the UV vacuum contributions also exist, which scales as

$$I_{11,UV} \propto \exp[-a_\eta mr] \quad (C23)$$

due to the singularity at $k = a_\eta m$ in the mode functions $u_A^{\text{WKB}}, u_B^{\text{WKB}}$. Thus we have shown that the diagonal element of Eq. (C6) is always exponentially suppressed.

Next, we turn to look at the off diagonal element I_{12}. Unlike the I_{11} case, whose integrand $|u_A|^2$ has a constant asymptotic value in the IR region, the I_{12}'s IR contribution

$$u_{A,k,\eta}u_{B,k,\eta}^* = \alpha_k \beta_k^* e^{-2i\int^\eta \omega d\eta'} \quad (C24)$$

contains e^{-2imt} time dependence. Physically, if we decompose the in state into the WKB vacuum and excitation state

$$|\text{in},\text{vac}\rangle = \sim|\text{WKB},\text{vac}\rangle + \sim|\text{WKB},\text{2-particles}\rangle + \sim|\text{WKB},\text{4-particles}\rangle, \quad (C25)$$

then this term comes from the interference term

$$\langle\text{WKB},\text{vac}|\psi_x\bar{\psi}_y|\text{WKB},\text{2-particles}\rangle \in \langle\text{in},\text{vac}|\psi_x\bar{\psi}_y|\text{in},\text{vac}\rangle. \quad (C26)$$

If we care about r large enough, for example corresponding to the CMB observation scale at recombination, we may assume the relevant k scale exits the horizon and becomes nonrelativistic during inflation. Thus we may safely use the dS mode function to evaluate $I_{12,IR,\text{CMB}}$.

Recall that during the dS era, we have Eq. (A28), where we choose the end of inflation time t_e as the reference point. Thus

$$u_{A,k,\eta}u_{B,k,\eta}^* = \frac{1}{2\pi}e^{-2im(t-t_e)+2i\frac{m}{H}\ln(2k/a_e H)}\Gamma^2\left(\frac{1}{2}-i\frac{m}{H}\right). \quad (C27)$$

Performing the integral using the steepest descent, we find the leading contribution comes from the $k \sim 0$ singularity in $u_{A,k,\eta}u_{B,k,\eta}^*$. We note that the k dependent phase factor $e^{2i\frac{m}{H}\ln(2k/H)}$ cannot be absorbed by a redefinition of the mode functions $u_{A,k,\eta}, u_{B,k,\eta}$, since this phase factor depends on the relative phase of $u_{A,k,\eta}, u_{B,k,\eta}$ which is fixed by the Bunch-Davies initial condition.

Plugging in Eq. (C11), we have

$$2\pi^2 I_{12,IR} = -e^{-2im(t-t(r))+i\phi(\frac{m}{H})}r^{-3}\sqrt{\frac{2\pi\frac{m}{H}}{\sinh(2\pi\frac{m}{H})}\left(1+\left(\frac{m}{H}\right)^2\right)}, \quad (C28)$$

where $\phi(\frac{m}{H}) = \text{Arg}(\Gamma(2+ix)\Gamma(\frac{1}{2}-ix))$ and $t(r)$ is the time when $a(t_r)Hr = 4$. We may consider the light mass limit

$$2\pi^2 I_{12,IR,\text{light}} \approx -e^{-2im(t-t(r))}r^{-3} \quad (C29)$$

and the heavy mass limit

$$2\pi^2 I_{12,IR,\text{heavy}} \approx -(4\pi)^{\frac{1}{2}}\left(\frac{m}{H}\right)^{\frac{3}{2}}\exp\left(-\pi\frac{m}{H}\right)e^{-2im(t-t(r))}r^{-3}. \quad (C30)$$

We may also consider the effect of having an IR cutoff k_{IR}, which is the scale that exits the horizon at the beginning of inflation. Such an IR cutoff will introduce a $\exp(-k_{\text{IR}}r)$ type of exponential suppression factor. However, for an observable universe with comoving radius R_{obs}, as long as $k_{\text{IR}}R_{\text{obs}} \ll 1$, we may ignore this suppression factor.

After evaluating the matrix element for the fermion correlators, we find that

(1) For the light fermion case, i.e. $m \ll H_{\text{inf}}$, in the limit $r \to \infty$

$$\langle\psi_x\bar{\psi}_y\rangle \approx \frac{1}{a_x^3}\frac{1}{2\pi^2}\begin{pmatrix} A & B \\ B^* & A \end{pmatrix}, \quad (C31)$$

where

$$A = \frac{1}{2}\pi a_*^3 \frac{m^2}{a_* r}\exp\left[-\sqrt{\frac{\pi}{2}}a_* mr\right]\cos\left(\sqrt{\frac{\pi}{2}}a_* mr\right), \tag{C32}$$

$$B = -i\hat{r}\cdot\vec{\sigma}e^{-2im(t-t_r)}r^{-3}, \tag{C33}$$

where a_* is evaluated at η_*.

(2) For the heavy fermion case, i.e. $m \gg H_{\rm inf}$, in the limit $r \to \infty$, we find in Eq. (C31)

$$A = \frac{1}{16}\sqrt{\pi}a_e^3(mH_e)^{\frac{3}{2}}\exp\left[-\frac{4m}{H_e} - \frac{1}{16}a_e^2 mH_e r^2\right], \tag{C34}$$

$$B = -i\hat{r}\cdot\vec{\sigma}(4\pi)^{\frac{1}{2}}\left(\frac{m}{H_e}\right)^{\frac{3}{2}}\exp\left(-\pi\frac{m}{H_e}\right)e^{-2im(t-t(r))}r^{-3}, \tag{C35}$$

and a_e is evaluated at the end of inflation.

Finally, we plug in the field correlator to $\langle n_{\psi,x}n_{\psi,y}\rangle$ and drop the terms that are exponentially suppressed when $r\to\infty$, to get Eq. (52).

APPENDIX D: RELATIVE SUPPRESSION OF COMMUTATORS

In this subsection, we want to compare the dependence on the scale factor $a(t)$ between $\langle in|[O_x,O_y]|in\rangle$ and $\langle in|\{O_x,O_y\}|in\rangle$, where O_x is a bosonic Hermitian operator and x, y are spacetime points located near the end of inflation. For simplicity, we take H as a constant. In particular, we are interested in the cases where $O = \sigma, \bar{\psi}\psi, \zeta$. We want to show that the commutator of O suffers from an additional suppression factor compared to the anticommutator.

In general, the diagonal matrix elements of products of the Hermitian operator obeys

$$(\langle in|O_xO_y|in\rangle)^* = \langle in|O_yO_x|in\rangle; \tag{D1}$$

therefore

$$\langle in|[O_x,O_y]|in\rangle = 2i\mathrm{Im}\langle in|O_xO_y|in\rangle, \tag{D2}$$

$$\langle in|\{O_x,O_y\}|in\rangle = 2\mathrm{Re}\langle in|O_xO_y|in\rangle. \tag{D3}$$

We can just study $\langle in|O_xO_y|in\rangle$. We may use the mode expansion of the field operator to evaluate such an expression and focus on modes that are outside of the horizon at both times η_x, η_y.

We shall first take $O=\sigma$, and we assume that the scalar is light, i.e. $m_\sigma < \frac{3}{2}H$, such that ν is real:

$$\langle in|\sigma_x\sigma_y|in\rangle = \int 4\pi k^2 dk \frac{[\int d^2\hat{k}e^{i\vec{k}\cdot(\vec{x}-\vec{y})}]}{(2\pi)^3 a_x^{3/2}a_y^{3/2}}\frac{1}{H}\frac{\pi}{4} \times [J_xJ_y + Y_xY_y + i(Y_xJ_y - J_xY_y)], \tag{D4}$$

where $J_x = J_\nu(\frac{k}{a_xH})$, $Y_x = Y_\nu(\frac{k}{a_xH})$ are the first and second kinds of Bessel functions with real values. The $d^2\hat{k}$ is the angular integral with normalization $\int d^2\hat{k} = 1$, and $\int d^2\hat{k}e^{i\vec{k}\cdot(\vec{x}-\vec{y})} = \sin(kr)/kr$ is real. If we focus on the k modes that are outside of the horizon, i.e. $k/aH \ll 1$, we may use the small argument expansion of the Bessel function, i.e. when $(0 < z < \sqrt{1+\nu})$

$$J_\nu(z) \approx \frac{1}{\Gamma(\alpha+1)}\left(\frac{z}{2}\right)^\nu, \tag{D5}$$

$$Y_\nu(z) \approx -\frac{\Gamma(\alpha)}{\pi}\left(\frac{2}{z}\right)^\nu. \tag{D6}$$

Then, under the common scaling of $a_x \to \lambda a_x$, $a_y \to \lambda a_y$, with λ increasing, we see the various terms in the correlator scales as

$$a_x^{-3/2}a_y^{-3/2}J_xJ_y \propto \lambda^{-2\nu-3}, \tag{D7}$$

$$a_x^{-3/2}a_y^{-3/2}Y_xY_y \propto \lambda^{2\nu-3}, \tag{D8}$$

$$a_x^{-3/2}a_y^{-3/2}(Y_xJ_y - J_xY_y) \propto \lambda^{-3}. \tag{D9}$$

Thus, we see under this common scaling, the IR contribution to the two point functions are

$$\langle in|\{\sigma_x,\sigma_y\}|in\rangle_{\rm IR} = 2\int_{\rm IR} 4\pi k^2 dk \frac{[\int d^2\hat{k}e^{i\vec{k}\cdot(\vec{x}-\vec{y})}]}{(2\pi)^3 a_x^{3/2}a_y^{3/2}} \times\frac{1}{H}\frac{\pi}{4}(J_xJ_y + Y_xY_y) \propto \lambda^{2\nu-3}, \tag{D10}$$

$$\langle in|[\sigma_x,\sigma_y]|in\rangle_{\rm IR} = 2i\int_{\rm IR} 4\pi k^2 dk \frac{[\int d^2\hat{k}e^{i\vec{k}\cdot(\vec{x}-\vec{y})}]}{(2\pi)^3 a_x^{3/2}a_y^{3/2}} \times\frac{1}{H}\frac{\pi}{4}(Y_xJ_y - J_xY_y) \propto \lambda^{-3}. \tag{D11}$$

Thus, we have shown under the scaling $a \to \lambda a$, the commutator of σ is suppressed by the $\lambda^{-2\nu}$ factor relative to its anticommutator. For the small mass scalar, $\lambda^{-2\nu} \approx \lambda^{-3+\frac{2m^2}{3H^2}}$.

For the case of $O=\zeta$, we have similar statements as the scalar case with $\nu = \frac{3}{2}$, i.e. $\langle[\zeta_x,\zeta_y]\rangle_{\rm IR}$ is suppressed by λ^{-3} relative to $\langle\{\zeta_x,\zeta_y\}\rangle_{\rm IR}$ under the scaling of $a\to\lambda a$.

Next, we consider the case of $O = \bar{\psi}\psi$. Using the mode decomposition Eq. (A30) and mode functions Eqs. (A23) and (A24), we have

$$\langle\bar{\psi}\psi_x\bar{\psi}\psi_y\rangle = \sum_{i,j}\frac{1}{a_x^3a_y^3}\frac{e^{i(\vec{k}_i+\vec{k}_j)\cdot(\vec{x}-\vec{y})}}{(2\pi)^6}[h_i^T(i\sigma_2)h_j] \times [h_j^\dagger(-i\sigma_2)h_i^*]F_{ij,x}F_{ij,y}^*, \tag{D12}$$

where

$$F_{ij,x} = r_i u_{B,i,x}u_{A,j,x} + (i \leftrightarrow j), \tag{D13}$$

$$F_{ij,x}F_{ij,y}^* = 2[r_i u_{B,i,x}u_{A,j,x} + (i \leftrightarrow j)](r_i u_{B,i,y}^* u_{A,j,y}^*) \tag{D14}$$

$$= 2[u_{B,i,x}u_{A,j,x}u_{B,i,y}^*u_{A,j,y}^* + r_ir_ju_{B,i,x}u_{A,j,x}u_{B,j,y}^*u_{A,i,y}^*]. \tag{D15}$$

We note that in Eq. (D12), the factor $e^{i(\vec{k}_i+\vec{k}_j)\cdot(\vec{x}-\vec{y})}$ after the angular average is real, and the factor $[h_i^T(i\sigma_2)h_j][h_j^\dagger(-i\sigma_2)h_i^*] = |[h_i^T(i\sigma_2)h_j]|^2$ is also real; thus the imaginary and real parts of $F_{ij,x}F_{ij,y}^*$ correspond to the commutator and anticommutator, respectively.

Next, we consider the two terms in Eq. (D15) one by one, using the explicit expression of Eq. (A29) to get

$$u_{B,i,x}u_{A,j,x}u_{B,i,y}^*u_{A,j,y}^* = \sqrt{\frac{\pi}{4}\frac{k_i}{a_xH}}\sqrt{\frac{\pi}{4}\frac{k_j}{a_xH}}\sqrt{\frac{\pi}{4}\frac{k_i}{a_yH}}\sqrt{\frac{\pi}{4}\frac{k_j}{a_yH}} \times (J_{+,i,x}+iY_{+,i,x}) \times (J_{-,j,x}+iY_{-,j,x})(J_{-,i,y}-iY_{-,i,y}) \times (J_{+,j,y}-iY_{+,j,y}), \tag{D16}$$

where

$$J_{\pm,i,x} = J_{\frac{1}{2}\pm i\frac{m}{H}}\left(\frac{k_i}{a_xH}\right), \qquad Y_{\pm,i,x} = Y_{\frac{1}{2}\pm i\frac{m}{H}}\left(\frac{k_i}{a_xH}\right). \tag{D17}$$

Using the small z expansion of the Bessel function again, where $\mathrm{Re}(\nu) = \frac{1}{2}$ in all the cases, we can extract its scaling behavior under $a \to \lambda a$,

$$\begin{aligned}(J_{+,i,x}&+iY_{+,i,x})(J_{-,j,x}+iY_{-,j,x})(J_{-,i,y}-iY_{-,i,y})(J_{+,j,y}-iY_{+,j,y})\\ &= Y_{+,i,x}Y_{-,j,x}Y_{-,i,y}Y_{+,j,y}\cdots\cdots \propto \lambda^2, \text{real}\\ &\quad - iJ_{+,i,x}Y_{-,j,x}Y_{-,i,y}Y_{+,j,y} - iY_{+,i,x}J_{-,j,x}Y_{-,i,y}Y_{+,j,y}\cdots\cdots \propto \lambda^1, \text{imaginary}\\ &\quad + iY_{+,i,x}Y_{-,j,x}J_{-,i,y}Y_{+,j,y} + iY_{+,i,x}Y_{-,j,x}Y_{-,i,y}J_{+,j,y}\cdots\cdots \propto \lambda^1, \text{imaginary}\\ &\quad + \text{terms subdominant in } \lambda \text{ expansion.}\end{aligned} \tag{D18}$$

Thus the imaginary part is suppressed by λ^{-1} relative to the real part. We can do a similar analysis on the second part $r_ir_ju_{B,i,x}u_{A,j,x}u_{B,j,y}^*u_{A,i,y}^*$ in Eq. (D15) and find the same behavior. Thus, for the $\bar{\psi}\psi$ operator, we have the following scaling law:

$$\langle\{\bar{\psi}\psi_x, \bar{\psi}\psi_y\}\rangle_{\mathrm{IR}} \propto \lambda^{-6}, \tag{D19}$$

$$\langle[\bar{\psi}\psi_x, \bar{\psi}\psi_y]\rangle_{\mathrm{IR}} \propto \lambda^{-7}. \tag{D20}$$

Thus, we see the commutator for $\bar{\psi}\psi$ gives additional suppression of the a^{-1} factor compared with the anticommutator, whereas the commutator for σ and ζ gives additional suppression o the a^{-3} factor.

APPENDIX E: EXPLICIT CHECK OF THE MASS INSERTION FORMULA

In this section, we show that the particle production part of the following equation holds using the adiabatic subtraction:

$$-i\int^y (dw)\langle[\bar{\psi}\psi_x, \bar{\psi}\psi_z]\rangle = \partial_m\langle\bar{\psi}\psi_x\rangle = \partial_m n_\Psi(x). \tag{E1}$$

Expressing both sides of Eq. (E1) using the mode sum, we see the left hand side is

$$-i\int^y (dw)\langle[\bar{\psi}\psi_x, \bar{\psi}\psi_w]\rangle = \frac{16}{a_x^3}\int^{y^0} dw^0 a_w\int\frac{d^3k}{(2\pi)^3} \times \mathrm{Im}[(u_{A,k}u_{B,k})_x(u_{A,k}u_{B,k})_w^*] \tag{E2}$$

and the right hand side is

$$\partial_m\langle\bar{\psi}\psi_x\rangle = \frac{2}{a_x^3}\int\frac{d^3k}{(2\pi)^3}\partial_m(|u_B|^2 - |u_A|^2). \tag{E3}$$

Thus, we only need to check for each given k, and the following equation is right:

$$\partial_m(|u_B|^2-|u_A|^2)=8\int^{y^0}dw^0a_w\mathrm{Im}[(u_{A,k}u_{B,k})_x(u_{A,k}u_{B,k})^*_w]. \tag{E4}$$

From the left hand side, we have

$$\partial_m(|u_B|^2-|u_A|^2)=-2\mathrm{Re}\left[(\,u_A^*u_B^*\,)\sigma_3\frac{\partial}{\partial m}\begin{pmatrix}u_A\\u_B\end{pmatrix}_{k,x}\right], \tag{E5}$$

and upon expressing the mode function at time x^0 in terms of the volution operator acting on the initial value, we have

$$\frac{\partial}{\partial m}\begin{pmatrix}u_A\\u_B\end{pmatrix}_{k,x}=-i\int_{\eta_i}^{x^0}dz^0U(x^0\leftarrow z^0)\frac{\partial}{\partial m}\begin{pmatrix}am & k\\k & -am\end{pmatrix}\times U(z^0\leftarrow\eta_i)\begin{pmatrix}u_A\\u_B\end{pmatrix}_{k,i}. \tag{E6}$$

Combining these two expressions, we can obtain the desired result after some algebra.

However, the remaining d^3k integrals in Eqs. (E2) and (E3) are UV divergent. To make them finite, we express both sides in terms of Bogoliubov coefficients and drop the pure vacuum contribution to get

$$-i\int^{x^0}(dw)\langle[\bar\psi\psi_x,\bar\psi\psi_w]\rangle\approx16\int\frac{d^3k}{(2\pi a_x)^3}\left(\frac{am}{\omega_k}\right)_x\times\int^x d\eta_wa_w\left(\frac{am}{\omega}\right)_w\mathrm{Im}[(\alpha\beta)_x(\alpha\beta)^*_w], \tag{E7}$$

$$\partial_m\langle\bar\psi\psi_x\rangle\approx\frac{2}{a_x^3}\int\frac{d^3k}{(2\pi)^3}\partial_m\left[2|\beta_{k,x}|^2\frac{a_xm}{\omega_{k,x}}\right]\approx\frac{4}{a_x^3}\int\frac{d^3k}{(2\pi)^3}\left(\frac{a_xm}{\omega_{k,x}}\right)\partial_m|\beta_{k,x}|^2. \tag{E8}$$

Now, we only need to check

$$\partial_m|\beta_{k,x}|^2=4\int^x d\eta_wa_w\left(\frac{am}{\omega}\right)_w\mathrm{Im}[(\alpha\beta)_x(\alpha\beta)^*_w]. \tag{E9}$$

Suppose x^0 is late enough such that $\beta_{k,x}$ is constant and equals its value at asymptotic future β_k; then we get

$$\partial_m|\beta_k|^2=4\int_{\eta_i}^{x^0}dz^0a_z\frac{am}{\omega}\mathrm{Im}(\alpha_k\beta_k)_x(\alpha\beta)^*_z. \tag{E10}$$

Thus, Eq. (E1) is compatible with the Bogoliubov projection.

APPENDIX F: GRAVITATIONAL INTERACTION

Here we derive the gravitational interaction. Consider the action

$$S=S_{EH}+S_\phi+S_\sigma+S_\psi \tag{F1}$$

$$=\int(dx)\left\{\frac{1}{2}M_p^2R+\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)\right]+\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma-\frac{1}{2}m_\sigma^2\sigma^2\right]+\bar\psi(i\gamma^\mu\nabla_\mu-m_\psi)\psi-\lambda\sigma\bar\psi\psi\right\}, \tag{F2}$$

where $M_p^2=\frac{1}{8\pi G}=1$. The metric is given in Arnowitt-Deser-Misner formalism[20] [95] by

$$g_{\mu\nu}=\begin{pmatrix}-N^2+h_{ij}N^iN^j & h_{ij}N^j\\h_{ij}N^j & h_{ij}\end{pmatrix},\quad g^{\mu\nu}=\begin{pmatrix}-N^{-2} & N^iN^{-2}\\N^iN^{-2} & h^{ij}-N^iN^jN^{-2}\end{pmatrix}, \tag{F3}$$

where h_{ij} is the metric tensor on the constant time hypersurface, and h^{ij} is the inverse metric. We use Latin indices $i,j,\ldots$, for objects on the three-dimensional constant time hypersurface, and we use h_{ij} and h^{ij} to raise and lower the indices. Then we use the Hamiltonian and the momentum constraints to determine the lapse function N and the shift vector N^i:

$$0=\frac{1}{N}\left[R^{(3)}-\frac{1}{N^2}(E_{ij}E^{ij}-E^2)\right]-2NT^{00}, \tag{F4}$$

$$0=\frac{2}{N}\nabla_i^{(3)}\left[\frac{1}{N}(E^{ij}-Eh^{ij})\right]+2N^jT^{00}+2T^{0j}, \tag{F5}$$

where $T^{\mu\nu}$ is the total matter stress tensor, $R^{(3)}$ is the Ricci scalar calculated with the three-metric h_{ij}, and

$$E_{ij}=\frac{1}{2}(\dot h_{ij}-\nabla_i^{(3)}N_j-\nabla_j^{(3)}N_i), \tag{F6}$$

$$E=E_{ij}h^{ij}. \tag{F7}$$

In order to consider the perturbation around the background configuration

[20] We use $(-+++)$ sign convention for the metric, and physical time t.

$$\phi^{(0)} = \bar{\phi}(t), \qquad \sigma^{(0)} = 0, \qquad g^{(0)}_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & a^2(t)\delta_{ij} \end{pmatrix}, \tag{F8}$$

where the background fields satisfy the background equations of motion

$$3H^2 = \frac{1}{2}\dot{\bar{\phi}}^2 + V(\bar{\phi}), \tag{F9}$$

$$\dot{H} = -\frac{1}{2}\dot{\bar{\phi}}^2, \tag{F10}$$

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V'(\bar{\phi}) = 0, \tag{F11}$$

we choose the comoving gauge, defined by[21]

$$\delta\phi = 0, \qquad \gamma_{ii} = 0, \qquad \partial_i\gamma_{ij} = 0, \tag{F12}$$

where

$$h_{ij} = a^2(t)[e^{\Gamma}]_{ij}, \qquad \Gamma_{ij} = 2\zeta\delta_{ij} + \gamma_{ij}. \tag{F13}$$

Then we solve the constraint equations (F4) and (F5) perturbatively using ζ and γ, and putting their solutions for N and N^i back into the action, we can get the perturbed action

[21]In this section, Latin indices i, j are raised and lowered by δ_{ij}, and repeated indices are contracted.

$$S^{(C)} = S^{(C)}_{\zeta\zeta} + S^{(C)}_{\sigma\sigma} + S^{(C)}_{\psi\psi} + S^{(C)}_{\gamma\gamma} + S^{(C)}_{\zeta\zeta\zeta} + S^{(C)}_{\zeta\sigma\sigma} + S^{(C)}_{\zeta\psi\psi} + S^{(C)}_{\zeta\sigma\sigma}\cdots. \tag{F14}$$

For the interaction terms $S^{(C)}_{\zeta\sigma\sigma}$ and $S^{(C)}_{\zeta\psi\psi}$, we need the solutions of N and N^i up to linear order in ζ,

$$N^{(1,C)} = 1 + \frac{\dot{\zeta}}{H}, \qquad N^{(1,C)}_i = \partial_i\left[-\frac{\zeta}{H} + \epsilon\frac{a^2}{\nabla^2}\dot{\zeta}\right], \tag{F15}$$

where $\epsilon \equiv \dot{H}/H^2$. Hence, the metric perturbations become

$$\delta g^{(C)}_{\mu\nu} = \begin{pmatrix} -2\frac{\dot{\zeta}}{H} & (-\frac{\zeta}{H} + \epsilon\frac{a^2}{\nabla^2}\dot{\zeta})_{,i} \\ (-\frac{\zeta}{H} + \epsilon\frac{a^2}{\nabla^2}\dot{\zeta})_{,i} & a^2(\delta_{ij}2\zeta + \gamma_{ij}) \end{pmatrix}, \tag{F16}$$

and we have the ζ-matter cubic interaction action

$$S^{(C)}_{\zeta\sigma\sigma} + S^{(C)}_{\zeta\psi\psi} = \frac{1}{2}\int d^4x\sqrt{-g}(T^{\mu\nu}_\sigma + T^{\mu\nu}_\psi)\delta g^{(C)}_{\mu\nu}, \tag{F17}$$

where $T^{\mu\nu}_\sigma$ and $T^{\mu\nu}_\psi$ are the stress energy tensors for σ and ψ, respectively, which are written as

$$T^{\mu\nu}_\sigma = g^{\mu\alpha}g^{\nu\beta}\partial_\alpha\sigma\partial_\beta\sigma + g^{\mu\nu}\mathcal{L}_\sigma, \tag{F18}$$

$$T^{\mu\nu}_\psi = -\frac{i}{2}[\bar{\psi}\gamma^{(\mu}\nabla^{\nu)}\psi - \nabla^{(\mu}(\bar{\psi})\gamma^{\nu)}\psi] + g^{\mu\nu}\mathrm{Re}(\mathcal{L}_\psi). \tag{F19}$$

Particularly, up to the cubic interaction, $\mathcal{L}_{\rm int} = -\mathcal{H}_{\rm int}$. Thus $S^{(C)}_{\zeta\sigma\sigma} + S^{(C)}_{\zeta\psi\psi} = -\int dt H_{\zeta\sigma\sigma}(t) + H_{\zeta\psi\psi}(t)$.

[1] P. Ade *et al.* (Planck Collaboration), Planck 2013 results. XXII. Constraints on inflation, Astron. Astrophys. **571**, A22 (2014).

[2] P. Ade *et al.* (Planck Collaboration), Planck 2013 results. XXIV. Constraints on primordial non-Gaussianity, Astron. Astrophys. **571**, A24 (2014).

[3] G. Hinshaw *et al.*, Nine-year Wilkinson microwave anisotropy probe (WMAP) observations: Cosmological parameter results, Astrophys. J. Suppl. Ser. **208**, 19 (2013).

[4] E. Komatsu *et al.* (WMAP Collaboration), Seven-year Wilkinson microwave anisotropy probe (WMAP) observations: Cosmological interpretation, Astrophys. J. Suppl. Ser. **192**, 18 (2011).

[5] W. J. Percival, S. Cole, D. J. Eisenstein, R. C. Nichol, J. A. Peacock, A. C. Pope, and A. S. Szalay, Measuring the baryon acoustic oscillation scale using the SDSS and 2dFGRS, Mon. Not. R. Astron. Soc. **381**, 1053 (2007).

[6] D. J. Eisenstein *et al.* (SDSS Collaboration), Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies, Astrophys. J. **633**, 560 (2005).

[7] A. A. Starobinsky, A new type of isotropic cosmological models without singularity, Phys. Lett. **91B**, 99 (1980).

[8] K. Sato, First order phase transition of a vacuum and expansion of the Universe, Mon. Not. R. Astron. Soc. **195**, 467 (1981).

[9] A. D. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, Phys. Lett. **108B**, 389 (1982).

[10] V. F. Mukhanov and G. Chibisov, Quantum fluctuation and nonsingular universe (in Russian), JETP Lett. **33**, 532 (1981).

[11] A. Albrecht and P. J. Steinhardt, Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking, Phys. Rev. Lett. **48**, 1220 (1982).

[12] S. Hawking and I. Moss, Fluctuations in the inflationary universe, Nucl. Phys. **B224**, 180 (1983).

[13] A. H. Guth and S. Pi, Fluctuations in the New Inflationary Universe, Phys. Rev. Lett. **49**, 1110 (1982).
[14] A. A. Starobinsky, Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations, Phys. Lett. **117B**, 175 (1982).
[15] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Spontaneous creation of almost scale-free density perturbations in an inflationary universe, Phys. Rev. D **28**, 679 (1983).
[16] J. Silk and M. S. Turner, Double inflation, Phys. Rev. D **35**, 419 (1987).
[17] D. Polarski and A. A. Starobinsky, Isocurvature perturbations in multiple inflationary models, Phys. Rev. D **50**, 6123 (1994).
[18] D. Langlois, Correlated adiabatic and isocurvature perturbations from double inflation, Phys. Rev. D **59**, 123512 (1999).
[19] D. H. Lyth and D. Wands, Generating the curvature perturbation without an inflaton, Phys. Lett. B **524**, 5 (2002).
[20] K. Enqvist and M. S. Sloth, Adiabatic CMB perturbations in pre–big bang string cosmology, Nucl. Phys. **B626**, 395 (2002).
[21] T. Moroi and T. Takahashi, Effects of cosmological moduli fields on cosmic microwave background, Phys. Lett. B **522**, 215 (2001).
[22] D. Seckel and M. S. Turner, Isothermal density perturbations in an axion dominated inflationary universe, Phys. Rev. D **32**, 3178 (1985).
[23] J. Preskill, M. B. Wise, and F. Wilczek, Cosmology of the invisible axion, Phys. Lett. **120B**, 127 (1983).
[24] L. Abbott and P. Sikivie, A cosmological bound on the invisible axion, Phys. Lett. **120B**, 133 (1983).
[25] M. Dine and W. Fischler, The not so harmless axion, Phys. Lett. **120B**, 137 (1983).
[26] P. J. Steinhardt and M. S. Turner, Saving the invisible axion, Phys. Lett. **129B**, 51 (1983).
[27] M. S. Turner, Cosmic and local mass density of invisible axions, Phys. Rev. D **33**, 889 (1986).
[28] A. D. Linde and V. F. Mukhanov, Non-Gaussian isocurvature perturbations from inflation, Phys. Rev. D **56**, R535 (1997).
[29] D. J. Chung, E. W. Kolb, and A. Riotto, Superheavy dark matter, Phys. Rev. D **59**, 023501 (1998).
[30] D. J. Chung, E. W. Kolb, A. Riotto, and L. Senatore, Isocurvature constraints on gravitationally produced superheavy dark matter, Phys. Rev. D **72**, 023511 (2005).
[31] D. J. Chung and H. Yoo, Isocurvature perturbations and non-Gaussianity of gravitationally produced nonthermal dark matter, Phys. Rev. D **87**, 023516 (2013).
[32] L. Kofman, G. R. Blumenthal, H. Hodges, and J. R. Primack, Generation of non-flat and non-Gaussian perturbations from inflation, ASP Conf. Ser. **15**, 339 (1991).
[33] N. Bartolo, S. Matarrese, and A. Riotto, Non-Gaussianity from inflation, Phys. Rev. D **65**, 103505 (2002).
[34] M. Kawasaki, K. Nakayama, T. Sekiguchi, T. Suyama, and F. Takahashi, Non-Gaussianity from isocurvature perturbations, J. Cosmol. Astropart. Phys. 11 (2008) 019.
[35] D. Langlois, F. Vernizzi, and D. Wands, Non-linear isocurvature perturbations and non-Gaussianities, J. Cosmol. Astropart. Phys. 12 (2008) 004.
[36] D. H. Lyth, Non-Gaussianity and cosmic uncertainty in curvaton-type models, J. Cosmol. Astropart. Phys. 06 (2006) 015.
[37] Z. Lalak, D. Langlois, S. Pokorski, and K. Turzynski, Curvature and isocurvature perturbations in two-field inflation, J. Cosmol. Astropart. Phys. 07 (2007) 014.
[38] M.-x. Huang, G. Shiu, and B. Underwood, Multifield DBI inflation and non-Gaussianities, Phys. Rev. D **77**, 023511 (2008).
[39] M. Beltrán, Isocurvature, non-Gaussianity, and the curvaton model, Phys. Rev. D **78**, 023530 (2008).
[40] X. Chen, Primordial non-Gaussianities from inflation models, Adv. Astron. **2010**, 638979 (2010).
[41] D. Langlois and T. Takahashi, Primordial trispectrum from isocurvature fluctuations, J. Cosmol. Astropart. Phys. 02 (2011) 020.
[42] D. Langlois and A. Lepidi, General treatment of isocurvature perturbations and non-Gaussianities, J. Cosmol. Astropart. Phys. 01 (2011) 008.
[43] D. Mulryne, S. Orani, and A. Rajantie, Non-Gaussianity from the hybrid potential, Phys. Rev. D **84**, 123527 (2011).
[44] J.-O. Gong and H. M. Lee, Large non-Gaussianity in non-minimal inflation, J. Cosmol. Astropart. Phys. 11 (2011) 040.
[45] M. Kawasaki, T. Kobayashi, and F. Takahashi, Non-Gaussianity from curvatons revisited, Phys. Rev. D **84**, 123506 (2011).
[46] M. Kawasaki, F. Takahashi, and T. Yanagida, Gravitino overproduction in inflaton decay, Phys. Lett. B **638**, 8 (2006).
[47] J. Pradler and F. D. Steffen, Constraints on the reheating temperature in gravitino dark matter scenarios, Phys. Lett. B **648**, 224 (2007).
[48] M. Endo, F. Takahashi, and T. Yanagida, Inflaton decay in supergravity, Phys. Rev. D **76**, 083509 (2007).
[49] T. Takahashi, M. Yamaguchi, J. Yokoyama, and S. Yokoyama, Gravitino dark matter and non-Gaussianity, Phys. Lett. B **678**, 15 (2009).
[50] M. Sasaki and E. D. Stewart, A general analytic formula for the spectral index of the density perturbations produced during inflation, Prog. Theor. Phys. **95**, 71 (1996).
[51] D. J. Chung, L. L. Everett, H. Yoo, and P. Zhou, Gravitational fermion production in inflationary cosmology, Phys. Lett. B **712**, 147 (2012).
[52] V. Kuzmin and I. Tkachev, Matter creation via vacuum fluctuations in the early universe and observed ultrahigh-energy cosmic ray events, Phys. Rev. D **59**, 123006 (1999).
[53] D. J. H. Chung, H. Yoo, and P. Zhou, Quadratic isocurvature cross-correlation, Ward identity, and dark matter, Phys. Rev. D **87**, 123502 (2013).
[54] N. Birrell and P. Davies, *Quantum Fields in Curved Space*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, UK, 1984).
[55] P. Peebles, *The Large-Scale Structure of the Universe*, Princeton Series in Physics (Princeton University Press, Princeton, NJ, 1980).
[56] G. Efstathiou and J. R. Bond, Isocurvature cold dark matter fluctuations, Mon. Not. R. Astron. Soc. **218**, 103 (1986).

[57] A. R. Liddle and D. Lyth, *Cosmological Inflation and Large Scale Structure* (Cambridge University Press, Cambridge, UK, 2000).
[58] D. Green, M. Lewandowski, L. Senatore, E. Silverstein, and M. Zaldarriaga, Anomalous dimensions and non-Gaussianity, J. High Energy Phys. 10 (2013) 171.
[59] J. R. Ellis, J. L. Lopez, and D. V. Nanopoulos, Confinement of fractional charges yields integer charged relics in string models, Phys. Lett. B **247**, 257 (1990).
[60] K. Benakli, J. R. Ellis, and D. V. Nanopoulos, Natural candidates for superheavy dark matter in string and M theory, Phys. Rev. D **59**, 047301 (1999).
[61] A. Kusenko and M. E. Shaposhnikov, Supersymmetric Q balls as dark matter, Phys. Lett. B **418**, 46 (1998).
[62] T. Han, T. Yanagida, and R.-J. Zhang, Adjoint messengers and perturbative unification at the string scale, Phys. Rev. D **58**, 095011 (1998).
[63] G. Dvali, Infrared hierarchy, thermal brane inflation and superstrings as superheavy dark matter, Phys. Lett. B **459**, 489 (1999).
[64] K. Hamaguchi, Y. Nomura, and T. Yanagida, Long-lived superheavy dark matter with discrete gauge symmetries, Phys. Rev. D **59**, 063507 (1999).
[65] K. Hamaguchi, K. Izawa, Y. Nomura, and T. Yanagida, Long-lived superheavy particles in dynamical supersymmetry breaking models in supergravity, Phys. Rev. D **60**, 125009 (1999).
[66] C. Coriano, A. E. Faraggi, and M. Plumacher, Stable superstring relics and ultrahigh-energy cosmic rays, Nucl. Phys. **B614**, 233 (2001).
[67] H.-C. Cheng, K. T. Matchev, and M. Schmaltz, Radiative corrections to Kaluza-Klein masses, Phys. Rev. D **66**, 036005 (2002).
[68] G. Shiu and L.-T. Wang, D matter, Phys. Rev. D **69**, 126007 (2004).
[69] V. Berezinsky, M. Kachelriess, and M. Solberg, Supersymmetric superheavy dark matter, Phys. Rev. D **78**, 123535 (2008).
[70] T. W. Kephart and Q. Shafi, Family unification, exotic states and magnetic monopoles, Phys. Lett. B **520**, 313 (2001).
[71] T. W. Kephart, C.-A. Lee, and Q. Shafi, Family unification, exotic states and light magnetic monopoles, J. High Energy Phys. 01 (2007) 088.
[72] P. Fox, A. Pierce, and S. D. Thomas, Probing a QCD string axion with precision cosmological measurements, arXiv: hep-th/0409059.
[73] N. Bartolo, E. Komatsu, S. Matarrese, and A. Riotto, Non-Gaussianity from inflation: Theory and observations, Phys. Rep. **402**, 103 (2004).
[74] T. Takahashi, M. Yamaguchi, and S. Yokoyama, Primordial non-Gaussianity in models with dark matter isocurvature fluctuations, Phys. Rev. D **80**, 063524 (2009).
[75] M. Kawasaki, K. Nakayama, T. Sekiguchi, T. Suyama, and F. Takahashi, A general analysis of non-Gaussianity from isocurvature perturbations, J. Cosmol. Astropart. Phys. 01 (2009) 042.
[76] S. Weinberg, *Cosmology* (Oxford University Press, New York, 2008).
[77] S. Weinberg, Ultraviolet divergences in cosmological correlations, Phys. Rev. D **83**, 063508 (2011).
[78] D. J. Chung, Classical inflation field induced creation of superheavy dark matter, Phys. Rev. D **67**, 083514 (2003).
[79] V. A. Kuzmin and I. I. Tkachev, Ultrahigh-energy cosmic rays and inflation relics, Phys. Rep. **320**, 199 (1999).
[80] D. J. Chung, P. Crotty, E. W. Kolb, and A. Riotto, On the gravitational production of superheavy dark matter, Phys. Rev. D **64**, 043503 (2001).
[81] S. Weinberg, Quantum contributions to cosmological correlations, Phys. Rev. D **72**, 043514 (2005).
[82] E. A. Calzetta and B.-L. Hu, *Nonequilibrium quantum field theory* (Cambridge University Press, Cambridge, UK, 2008).
[83] M. P. Hertzberg, M. Tegmark, and F. Wilczek, Axion cosmology and the energy scale of inflation, Phys. Rev. D **78**, 083507 (2008).
[84] E. W. Kolb and M. S. Turner, The early universe, Front. Phys. **69**, 1(1990).
[85] P. Sikivie, Axion cosmology, Lect. Notes Phys. **741**, 19 (2008).
[86] S. Weinberg, Quantum contributions to cosmological correlations, Phys. Rev. D **72**, 043514 (2005).
[87] E. Calzetta and B. Hu, *Nonequilibrium Quantum Field Theory*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, UK, 2008).
[88] R. Bean, J. Dunkley, and E. Pierpaoli, Constraining isocurvature initial conditions with WMAP 3-year data, Phys. Rev. D **74**, 063503 (2006).
[89] E. Komatsu *et al.*, Seven-year Wilkinson microwave anisotropy probe (WMAP1) observations: Cosmological interpretation, Astrophys. J. Suppl. Ser. **192**, 18 (2011).
[90] D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. Nolta, C. Bennett, B. Gold, M. Halpern, R. Hill, and N. Jarosik, Seven-year Wilkinson microwave anisotropy probe (WMAP) observations: Power spectra and WMAP-derived parameters, Astrophys. J. Suppl. Ser. **192**, 16 (2011).
[91] E. Komatsu and D. N. Spergel, Acoustic signatures in the primary microwave background bispectrum, Phys. Rev. D **63**, 063002 (2001).
[92] C. Hikage, K. Koyama, T. Matsubara, T. Takahashi, and M. Yamaguchi, Limits on isocurvature perturbations from non-Gaussianity in WMAP temperature anisotropy, Mon. Not. R. Astron. Soc. **398**, 2188 (2009).
[93] C. Hikage, M. Kawasaki, T. Sekiguchi, and T. Takahashi, CMB constraint on non-Gaussianity in isocurvature perturbations, J. Cosmol. Astropart. Phys. 07 (2013) 007.
[94] C. Hikage, M. Kawasaki, T. Sekiguchi, and T. Takahashi, Extended analysis of CMB constraints on non-Gaussianity in isocurvature perturbations, J. Cosmol. Astropart. Phys. 03 (2013) 020.
[95] R. Arnowitt, S. Deser, and C. Misner, The dynamics of general relativity, in *Gravitation: An Introduction to Current Research*, edited by L. Witten (Wiley, New York, 1962), Chap. 7, pp. 227–265.