RESEARCH PLAN (2025)

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I am interested in symplectic geometry and representation theory, more generally theoretical physics inspired math. I worked on

(1) homological mirror symmetry and Fukaya category
(2) categorified quantum group via Fukaya category
(3) Perverse schober and factorizable perverse schober

I am listing below recent work and future directions.

1. Homological Mirror Symmetry

Mirror symmetry is a duality involving complex geometry and symplectic geometry. It has many aspects, SYZ torus fibration, quantum differential equation matching period integrals, the one that I care most is about equivalences of two kind of categories, called 'homological mirror symmetry', where on one side we have the category $Coh(X)$ of coherent sheaves on a complex variety X, and on the other side we have the Fukaya category $\mathrm{Fuk}(Y)$ of Lagrangians in a symplectic manifold Y

$$\text{(B-side)} \quad Coh(X) \simeq Fuk(Y) \quad \text{(A-side)}.$$

The beauty of such duality is that, one relates something rigid but easy to compute (B-side) to something flexible yet often hard to compute (A-side).

1.1. Toric CY VGIT, Schober and Windows.

The most thoroughly studied mirror symmetry is about toric variety. One way to obtain toric variety is by GIT quotient. We recall the setup.

Let $T = (\mathbb{C}^*)^k$ and let $T^\vee$ be the Pontryagin dual torus. Let $\beta_1, \cdots, \beta_N \in X^*(T) = X_*(T^\vee)$.

(1) On the B-side, we have a toric quotient stack $X = [\mathbb{C}^N/T]$, where T acts with weight β. We can consider various GIT quotient substacks $X_\chi = [\mathbb{C}^N//_\chi T]$ parametrized by character $\chi \in X^*(T)$.
(2) On the A-side, we have a Landau-Ginzburg model $W = z_1 + \cdots + z_N : (\mathbb{C}^*)^N \to \mathbb{C}$ with $\pi = (\beta_1, \cdots, \beta_N) : (\mathbb{C}^*)^N \to T^\vee$. We can consider the Fukaya category of various fibers $C_b = \mathrm{Fuk}(\pi^{-1}(b), W_b)$, where $b \in T^\vee$, $W_b = W|_{\pi^{-1}(b)}$.

In the case where X is Calabi-Yau, all the GIT quotient stacks X_χ are derived equivalent, and is equivalent to the A-side C_b for generic $b \in T^\vee$. More precisely, there is a descriminant locus $\Delta \subset T^\vee$, given by the singular value of $\pi|_{W^{-1}(0)}$. The data $((\mathbb{C}^*)^N, W, \pi)$ should give a perverse schober on $T^\vee$ with singularity along Δ.

In [HZ22a], we studied the certain numerical invariant associated to the descriminant loci, and matched it with B-side numerics from VGIT wall crossing. It would be interesting to match the actual categories associated to the discriminant / walls on both sides.

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Question 1. *What is this schober on $(T^\vee, \Delta)$, the nearby cycle categories on the complement of Δ, the vanishing cycle categories attached to Δ? How to relate this setup to 3d A-brane in* [CL24]*?*

A useful tool to study wall-crossing in VGIT is the window subcategory, which is a section of the restriction functor $\mathrm{Coh}(X) \to \mathrm{Coh}(X_\chi)$ from the full stack to the GIT quotient stack. On one hand, there exists general recipe for producing window subcategory [HL15]. For some simple toric CY cases, we can produce microlocal sheaf theoretic window on A-side [Zho, HZ22b]. However, it is unclear what is the mirror of window category on the A-side.

Question 2. *For a given B-side window category, can one construct its mirror window category on the A-side?*

Here is an approach: we can lift $T^\vee = (\mathbb{C}^*)^k$ to the cover$\mathbb{C}^k_x$, and lift Δ to $\widetilde{\Delta}$. If we find a fully faithful sectorial embedding $T^*[0,1]^k$ into $(\mathbb{C}^k_x \backslash \widetilde{\Delta}, \sum_i x_i^2)$, then this could induce a universal window.

Another question is related to non-commutative crepant resolution (NCCR) of the affine quotient $\mathbb{C}^N/T$. It is expected that a commutative resolution $\mathrm{Coh}(X_\chi)$ for generic χ admits a tilting generator, hence by mirror symmetry, the Fukaya category of a generic fiber $\mathrm{Fuk}(\pi^{-1}(b), W_b)$ should also admit a tilting generator Lagrangian.

Question 3. *Can one construct tilting generator for* $\mathrm{Fuk}(\pi^{-1}(b), W_b)$ *in a geometric way? What's the criterion for a Lagrangian to have endomorphism purely in degree* 0*?*

1.2. **Grassmannian and braid variety.** Marsh-Rietsch proposed a mirror symmetry conjecture

$$\mathrm{Coh}(Gr(n-k,n)) \simeq \mathrm{Fuk}(Gr(k,n)^o, W), \quad k \leq 2n.$$

where $Gr(k,n)^o$ is certain complement of anti-canonical divisor, more precisely the open positroid.

Despite that we know many things about $Gr(k,n)$, such as its Kapranov exceptional collection, the critical points for W, the cluster structures for $Gr(k,n)^o$, the Newton-Okounkov body and Gelfand-Zeitlin polytope and relation to toric degenerations, the proof of mirror symmetry remain elusive.

With Linhui Shen, Zhe Sun and Daping Weng, we made progress in the simplest case, by decompactifying the B-side and removing superpotential on the A-side. We proved

$$\mathrm{Coh}(Gr(2,n)^o) \simeq \mathrm{Fuk}(Gr(2,n)^o).$$

The proof uses the fact that $Gr(2,n)^o$'s cluster quiver is acylic, and we uses Deodhar decomposition to find B-side Zariski open cover, and A-side's mirror cover of the skeleton, and proved the HMS statement from Zariski descent.

With Joe Hlavinka, we are investigating

Question 4. *How to prove the original* $Gr(2,n)$ *mirror symmetry? How to generalize to* $Gr(3,n)^o$ *and* $Gr(3,n)$*? How does the mirror symmetry for* $\mathrm{Coh}(T^*Gr(k,n))$ *help us in understanding the mirror symmetry for* $\mathrm{Coh}(Gr(k,n))$*?*

1.3. **Mirror of Zariski descent.** One way to study $\mathrm{Coh}(X)$ is by Zariski descent, namely constructing open affine cover $X = \cup_i U_i$, then present $\mathrm{Coh}(X)$ as limit over the Cech diagram

$$\lim\Big(\prod_i \mathrm{Coh}(U_i) \to \prod_{i<j} \mathrm{Coh}(U_i \cap U_j) \to \cdots\Big).$$

If we can find mirror $\mathrm{Fuk}(Y_i)$ for each $\mathrm{Coh}(U_i)$ and present $\mathrm{Fuk}(Y)$ as a limit over a similar diagram, then we can prove mirror symmetry via gluing.

This approach has been adopted in Lee's thesis[Lee16] extended by Auroux-Smith [AS21] to study mirror symmetry for Riemann surfaces via pair-of-pants decomposition. With Hayato Morimura and Nicolò Sibilia [MSZ23], we generalize this method to higher dimension, and use pair-of-pants decomposition on complete intersections in $(\mathbb{C}^*)^n$ to match certain matrix factorization mirror.

The key technical ingredient is to understand the characteristic flow on the boundary of the local pieces. In the Riemanns surface case, each boundary circle has a neighborhood that embeds to cylinder T^*S^1, and flows are easily understood. In the higher dimensional case, we can also locally model the boundary by $(T^k_\theta \times \mathbb{R}^k_b) \times F$, hence if the boundary is cut-out by $f(b)$, then the flow is only along θ direction and static along the F factor. However, the gluing argument that we use requires Weinstein domain for all the pieces involved, whereas the argument in [Lee16, AS21] does not need that.

Question 5. *Can we generalize the argument of Lee-Auroux-Smith to higher dimensional pair-of-pants, and obtain local-to-global gluing without using Weinstein structure? Can we adapt the method to symmetric product of curve, and prove the conjecture in* [LP21].

2. Categorified Quantum Group via Fukaya category

Let $\mathfrak{g}$ be an ADE type Lie algebra. Mina Aganagic proposed two ways to categorify knot invariants and categorify quantum group [Aga20, Aga21]. With Mina Aganagic and collaborators [ADL+24], we developed a Fukaya category approach to categorify $U_q(\mathfrak{g})_-$ and relates Floer theoretic computation to quiver Hecke algebra developed by Khovanov-Lauda-Rouquier-Webster (KLRW).

Let r be the rank of $\mathfrak{g}$. Let w_i be the fundamental weights, and α_i be the simple roots, for $i = 1, \cdots, r$. We use V_{w_i} for the correponding highest weight representation. Let $\Phi_+ \simeq (\mathbb{Z}_{\geq 0})^r$ denote the set of positive roots. If $k = (k_1, \cdots, k_r) \in \Phi_+$, we denote $\mathrm{Sym}^k S = \prod_{i=1}^r \mathrm{Sym}^{k_i}(S)$.

2.1. FukSym category.

Let $\overline{S}$ be a smooth compact surface with boundary with boundary $\partial\overline{S}$ and interior S^o. Let $E = E_1 \sqcup \cdots \sqcup E_m \subset \partial\overline{S}$ be a collection of disjoint open intervals called **stops**, and let $P \subset S^o$ be a finite subset colored by miniscule weights, called **$\mathfrak{g}$-defects**. Let $S := (S^o \cup E, P)$, called a **decorated surface**. A closed embedding $\iota : S_1 \hookrightarrow S_2$ is an embedding of the underlying surface, with matching defects $P_1 = \iota^{-1}(P_2)$.

Given a decorated surface S, and given $k \in \Phi_+$, we can produce a Weinstein sector X_k with a map $X_k \to \mathrm{Sym}^k S$. We denote $\mathrm{FukSym}_\mathfrak{g}(S)_k = \mathrm{Fuk}(X_k)$[1] and let

$$\mathrm{FukSym}_\mathfrak{g}(S) = \oplus_{k\in\Phi_+} \mathrm{FukSym}_\mathfrak{g}(S)_k.$$

For $\mathfrak{g} = sl_2$, k ranges through $\mathbb{Z}_{\geq 0}$, and we can omit the coloring of P. From now on, we omit $\mathfrak{g}$ in the notation.

This assignment enjoys the following properties

(1) If $S = S_1 \sqcup S_2$, then $\mathrm{FukSym}(S) = \mathrm{FukSym}(S_1) \otimes \mathrm{FukSym}(S_2)$.
(2) If $S_1 \hookrightarrow S_2$ is a closed embedding, then we have functors $\mathrm{FukSym}(S_1)_k \to \mathrm{FukSym}(S_2)_k$.

[1] If $\mathfrak{g}$ were $\mathfrak{gl}_{1|1}$, then $\mathrm{FukSym}_\mathfrak{g}(S)_k = \mathrm{Fuk}(\mathrm{Sym}^k((S^o\backslash P) \cup I))$, i.e. Heegaard-Floer theory.

(3) Isotopy of stops and defects induces equivalences of categories.

From these properties, we can obtain monoidal categories and monoidal modules:

$$\mathcal{A} := \mathrm{FukSym}(T^*[-1,1])$$

with monoidal product given by

$$\mu : \mathcal{A} \otimes \mathcal{A} \simeq \mathrm{FukSym}(T^*[-1,-1/2] \sqcup T^*[1/2,1]) \to \mathrm{FukSym}(T^*[-1,1]) = \mathcal{A}.$$

And a decorated surface S with stops $I_1, \cdots, I_m$ makes $\mathrm{FukSym}(S)$ a module over $\mathcal{A}^{\otimes m}$.

Most importantly, such assignment satisfies cutting-gluing axioms for TFT in the following sense. Let S be a decorated surface, H an interval in S separating S to two parts $S_1^o \sqcup S_2^o = S \backslash H$. Let $S_i = S_i^o \cup H$ be decorated surfaces with H viewed as a stop, and let $\mathcal{A}$ acts on S_i at stop H. Then we prove

Theorem 6 ([SZ25])**.** $\mathrm{FukSym}(S) \simeq \mathrm{FukSym}(S_1) \otimes_{\mathcal{A}} \mathrm{FukSym}(S_2)$.

To connect to previous work on categorification, we proved that [ALSZ25]

(1) If S is a disk with one stop and 1 defects, then $\mathrm{FukSym}(S)$ categorifies $U_q(sl_2)$ representation $\mathbb{C}^2$, with $\mathrm{FukSym}(S)_0 = \mathrm{Vect}$ corresponding to the highest weight space, and $\mathrm{FukSym}(S)_1$ corresponding to lowering the weight once. $\mathrm{FukSym}(S)_{>1} = 0$.
(2) If S is a disk with one hole (i.e. S is an annulus) and one stop, then $\mathrm{FukSym}(S)$ categorifies the (universal) Verma representation of $U_q(sl_2)$.

One of the novelty here is that we have disk with 3 stops, which categorifies coproduct structure, i.e., a tri-module D_3 over $\mathcal{A}$. Given two right $\mathcal{A}$-modules M and N, and given the $(\mathcal{A}^{\otimes 2}, \mathcal{A})$-module D_3, we may form the tensor product

$$M \widetilde{\otimes} N := (M \otimes N) \otimes_{\mathcal{A} \otimes \mathcal{A}} D_3$$

which is again a right $\mathcal{A}$-module. This makes $\mathcal{A} - mod^R$ a monoidal category.

2.2. **Web functor.** We consider general $\mathfrak{g}$ here. Consider a decorated surface S times interval $[0,1]$ with varying defects in $S \times [0,1]$.

(1) When we isotope the defects, they sweep out a braid. Braid functor is an equivalence.
(2) We can produce a pair of defects, labelled by weight λ and $\lambda^* = -w_0\lambda$. This is called a cup. The cup functor categorifies $\mathbb{C} \to V_\lambda \otimes V_{\lambda^*}$. The adjoint of cup is cap, which pair-annihilate λ and λ^* labelled defects.
(3) More generally, we can have web functors categorifying intertwiner $\otimes_i V_{\lambda_i} \to \otimes V_{\mu_i}$.

These functors has been defined algebraically in [MW18]. However, apart from the braiding functors, we still need to understand how the more general web functor work in a symplectic way.

Question 7. *Can one understand the web functors geometrically?*

2.3. **Morphisms between Webs.** Our eventual goal is to construct a 2-functor from the 2-categor of defect/web/foams, to the 2-category of $\mathcal{A}$-mod, where $\mathcal{A} = \mathrm{FukSym}(T^*[0,1])$ is the KLR monoidal category. However, the definition of 2-morphism in the 2-category $\mathcal{A}$-mod is too cumbersome (think of Hochschild cohomology of a category in the case $\mathcal{A}$ is trivial), thus it is desirable to find a better description. Here we use a geometric realization of web functor as web Lagrangian.

Let D_m be a disk with m stops. Let P_1, P_2 be $\mathfrak{g}$-defects in D_1, let W be a $\mathfrak{g}$-web in $D_1 \times [0,1]$ connecting P_1 and P_2. Let $U(W)$ denote the 'bending up' of web W by putting the bottom of W

to the top right, and view $U(W)$ as web from $\emptyset$ to $P_2\#P_1^*$. Let $L_W \in \mathrm{FukSym}((D_0, P_2\#P_1^*))$ be the image of of the web functor $U(W)$ applied to the unit object $\mathbb{C}$ in $\mathrm{Vect} = \mathrm{FukSym}(D_0)_0$. We call $L(W)$ the **web Lagrangian.**

Question 8. *How to send a cobordism (foam) between webs W_1, W_2 to morphism between web Lagrangisn $L(W_1), L(W_2)$?*

2.4. **Circle Cut.** The simplest circle cut is Lee-Auroux-Smith's decomposition of Riemann surface. Given a surface S cut along some circles into a union of S_i (assuming no circle separating the same S_i), they have a limit diagram

$$\mathrm{Fuk}(S) \xrightarrow{\sim} \lim[\prod_i \mathrm{Fuk}(S_i) \to \prod_{i,j} \mathrm{Fuk}(S_i \cap S_j)],$$

where $S_i \cap S_j$ means the collar neighborhoods of the interface circle.

To get circle-cut gluing, we will consider co-monoidal category and co-modules. Let $C = S^1 \times (-1,1)$, $C_1 = S^1 \times (-1,0)$, $C_2 = S^1 \times (0,1)$. We expect there exists a co-multiplication map

$$\Delta : \mathcal{C} = \mathrm{FukSym}(C) \to \mathrm{FukSym}(C_1) \otimes \mathrm{FukSym}(C_2) \simeq \mathcal{C} \otimes \mathcal{C}.$$

And any cirlce boundary of a surface give a place for co-action of $\mathcal{C}$.

There should be a co-tensor formula for gluing along circles. Suppose C is a circle in S, cutting S into two connected components S_1 and S_2. Assume S_1 and S_2 are not disks, then we expect

$$\mathrm{FukSym}(S) \xrightarrow{\sim} \mathrm{FukSym}(S_1) \otimes^{\mathcal{C}} \mathrm{FukSym}(S_2)$$

where one can use limit diagram of cobar resolution to compute the categorical co-tensor.

Question 9. *Construct the co-monoidal category $\mathcal{C}$ using Fukaya category.*

In our actual application, we would need one of the surfaces after cut to be a disk with defects. For example, let $\mathfrak{g} = sl_2$, S be a disk with two defects (say near 0), and circle C encloses the two defects. Let $S \backslash C = S_1 \cup S_2$, where S_1 is the disk with a hole and a stop, and S_2 is the disk with no stop and two defects. Then we would expect, for $k = 1$,

$$\mathrm{FukSym}(S)_1 \xrightarrow{\sim} \lim[\mathrm{FukSym}(S_1)_1^{\alpha} \to \mathrm{FukSym}(C)_1^{\alpha} \leftarrow \mathrm{FukSym}(S_2)_1^{\alpha}]$$

where α subscript reminds us that there might be some categorical deformation.

Question 10. *Can one describe such categorical deformation explicitly? Can one do calculation with the above circle-cut description?*

3. Perverse Schober

These are joint work in progress with Yuji Okitani.

3.1. **Schober from presheaf on perverse sectors.** Perverse schober on a complex manifold X with a complex stratification S, denoted as $2Perv(X,S)$, should be a 2-category that categorifies the abelian category of perverse sheaves $Perv(X,S)$ [KS14].

Let $\Lambda \subset T^*X$ be the union of conormal to strata of S. Morally speaking, there should be a wrapped 2-Fukaya category (or Feuter category) $2\,\mathrm{Fuk}(\Lambda)$, where objects are holomorphic Lagrangians transverse to Λ (L transverse to Λ means $L \cap \Lambda$ compact). Then $2Perv(X,S)$ should be module over $2Fuk(\Lambda)$. However, it is hard to do wrapping of holomorphic Lagrangian, and also hard to solve Feuter equation.

Instead, we propose that $2Perv(X, S)$ is an assignment of certain testing object, called **perverse sector** to Cat, satisfying some conditions. We first define what is a perverse sector.

Definition. *A* **local LG-model** $f : (U, E_U) \to (B, E_B)$ *is the following data,* $U \subset X$ *an open ball,* $B \subset \mathbb{C}$ *an open ball,* $f : U \to B$ *holomorphic with finitely many atypical fiber,* $\emptyset \neq E_B \subset \partial B$ *a finite union of separated open intervals, and* $E_U = f^{-1}(E_B)$*. A* **perverse sector** (U, E_U) *is a an open ball* U *and a open subset* $E_U \subset \partial U$ *(with* ∂E_U *piecewise smooth) that (after a stratified isotopy) admits a local LG-model. We sometimes also call* $U \cup E_U$ *a perverse sector.*

We will omit the adjective perverse when there is no ambiguity.

If $D = (U, E_U)$ is a sector, then we call $D^\vee = (U, E_U^\vee = \partial U - \overline{E}_U)$ the dual sector. There are two isotopies R_+, R_- from D to $D^\vee$, where in the case of local LG-model, R_+ rotate E_B to $E_B^\vee$ CCW. If $i : D_1 \hookrightarrow D_2$ is a closed embedding, then $i^\vee : D_1^\vee \hookrightarrow D_2^\vee$ is an open embedding.

Definition. *A schober* P *is the following data*

(1) *For each sector* D*, we assign a category* $P(D)$*.*
(2) *For each closed embedding of sectors* $i : D_1 \hookrightarrow D_2$*, we assign a functor* $i_! : P(D_1) \to P(D_2)$*,*
(3) *For each open embedding of sectors* $j : D_1 \hookrightarrow J_2$*, we assign* $j^* : P(D_2) \to P(D_1)$*.*
(4) *For each isotopy* $\gamma : D_1 \rightsquigarrow D_2$*, we have an equivalence* $\gamma : P(D_1) \xrightarrow{\sim} P(D_2)$*.*

such that

(1) *If* $D = D_1 \sqcup D_2$*, then* $P(D) = P(D_1) \oplus P(D_2)$*,*
(2) $i_!$ *and* j^* *admit left and right adjoints infinitely many times. Concretely,* $i^! := R_+(i^\vee)^* R_-$ *is the right-adjoint of* $i_!$*, and* $j_* = R_+(i^\vee)^* R_-$ *is right-adjoint to* j_**.*
(3) *If* $D = D_1 \cup D_2$ *is an sector, and* D_1, D_2 *are closed sub-sector of* D*, then we require*
$$\mathrm{colim}(P(D_1 \cap D_2) \to P(D_1) \oplus P(D_2)) \xrightarrow{\sim} P(D).$$
(4) *If* $i : D_1 \hookrightarrow D$ *is an closed embedding and* $j : D_2 \hookrightarrow D$ *is an open embedding, such that* $D = D_1 \sqcup D_2$*, then we have SOD decomposition*
$$P(D) = \langle j_* P(D_2), i_! P(D_1) \rangle.$$

Question 11. *Show the geometric definition of schober agrees with existing combinatorial definition of schober.*

3.2. Factorizable Schober.

Let C be a smooth curve $S \subset C$ a finite set of points. By $\mathbb{N}$-graded category C, we mean a sequence of categories $C_0 = \mathrm{Vect}, C_1, C_2, \cdots$, and $C = \oplus_n C_n$. Functors between $\mathbb{N}$-graded category preserves grading.

Definition. *A factorizable perverse schober* P *on* (C, S) *is the same data as a perverse schober on* (C, S)*, except for each sector* D*, we assign a* $\mathbb{N}$*-graded category* $P(D)$*. The assignment needs to satisfy*

(1) *If* $D = D_1 \sqcup D_2$*, then* $P(D) = P(D_1) \otimes P(D_2)$*.*
(2) $i_!$ *and* j^* *admit left and right adjoints infinitely many times. Concretely,* $i^! := R_+(i^\vee)^* R_-$ *is the right-adjoint of* $i_!$*, and* $j_* = R_+(i^\vee)^* R_-$ *is right-adjoint to* j_**.*
(3) *If* D *is an sector, and* D_1, D_2 *are closed sub-sector of* D*,* $H = D_1 \cap D_2$ *a collar neighborhood of an interval, then we require*
$$P(D_1) \otimes_{P(H)} P(D_2) \xrightarrow{\sim} P(D).$$

(4) *If* $i : D_1 \hookrightarrow D$ *is an closed embedding and* $j : D_2 \hookrightarrow D$ *is an open embedding, such that* $D = D_1 \sqcup D_2$, *then we have SOD decomposition*

$$P(D)_n = \langle C_{n,0}, C_{n-1,1}, \cdots, C_{0,n} \rangle, \quad C_{k,n-k} \simeq P(D_2)_k \otimes P(D_1)_{n-k}$$

Question 12. *Show that the geometric definition of schober agrees with Dyckerhoff-Wedrich's factorizable schober on* $\mathbb{C}$.

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